GATEFREAKS

GATE/NET/PSU

COMPUTER SCIENCE

Discrete Mathematics and Theory of Computation

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Contents

6

Set Theory and Boolean Algebra

1. $A \vee A = A$ is called:
(A) Identity law
(B) De Morgan's law
(C) Idempotent law
(D) Complement law
UGCNET2004-II(dec.)

2. If $f(x) = x + 1$ and $g(x) = x + 3$ then $f \circ f \circ f \circ f$ is:
(A) g
(B) $g + 1$
(C) g^4
(D) None of the above
UGCNET2004-II(dec.)

3. Let A and B be two arbitrary events, then:
(A) P(A∩B) = P(A)P(B)
(B) P(A∪B) = P(A)+P(B)
(C) P(A∪B)≤ P(A)+P(B)
(D) P(A/B) = P(A∩B)+P(B)
UGCNET2005-II(june)

4. The transitive closure of a relation R on set A whose relation matrix
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is
(A) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$
UGCNET2005-II(dec.)

5. Let A = $\{x| - 1 < x < 1\}$ = B. The function $f(x) = x/2$ from A to B is:
(A) injective
(B) surjective
(C) both injective and surjective
(D) neither injective nor surjective
UGCNET2006-II(june)

6. The idempotent law in Boolean algebra says that:
(A) ¬(¬x)=x
(B) x+x=x
(C) x+xy=x
(D) x(x+y)=x
UGCNET2008-II(jun.)

7. A relation R in {1,2,3,4,5,6} is given by {(1,2),(2,3),(3,4),(4,4),(4,5)}. This relation is:
(A) reflexive
(B) symmetric
(C) transitive
(D) not reflexive, not symmetric and not transitive
UGCNET2008-II(dec.)

8. If she is my friend and you are her friend, then we are friends. Given this, the friend relationship in this context is ....
(i) commutative
(ii) transitive
(iii) implicative
(iv) equivalence
(A) (i) and (ii)
(B) (iii)
(C) (i), (ii), (iii) and (iv)
(D) None of these
UGCNET2009-II(dec.)

9. The number of integers between 1 and 250 that are divisible by 2, 5 and 7 is
(A) 2
(B) 3
(C) 5
(D) 8
UGCNET2010-II(dec.)

10. Domain and Range of the function $y = -\sqrt{2x + 3}$ is
(A) $x > \frac{3}{2}, y > 0$
(B) $x > \frac{3}{2}, y \leq 0$
(C) $x \geq \frac{3}{2}, y \leq 0$
(D) $x \leq \frac{3}{2}, y \leq 0$
UGCNET2011-II(Dec.)

11. How many relations are there on a set with n elements that are symmetric and a set with n elements that are reflexive and symmetric ?
(A) $2^{\frac{n(n+1)}{2}}$ and $2^n.3^{\frac{n(n-1)}{2}}$
(B) $3^{\frac{n(n-1)}{2}}$ and $2^{n(n-1)}$
(C) $2^{\frac{n(n+1)}{2}}$ and $3^{\frac{n(n-1)}{2}}$
(D) $2^{\frac{n(n+1)}{2}}$ and $2^{\frac{n(n-1}{2}}$
UGCNET2012-III(JUN)

12. The power set of the set $\{\Phi\}$ is
(A) $\{\Phi\}$
(B) $\{\Phi, \{\Phi\}\}$
(C) $\{0\}$
(D) $\{0, \Phi, \{\Phi\}\}$
UGCNET2012-II(Dec)

13. The power set of A∪B, where A = {2, 3, 5, 7} and B = {2, 5, 8, 9} is
(A) 256
(B) 64
(C) 16
(D) 4
UGCNET2012-III(Dec)

14. The relation "divides" on a set of positive integers is ....
(A) Symmetric and transitive
(B) Anti symmetric and transitive
(C) Symmetric only
(D) Transitive only
UGCNET2013-II(JUN)

15. How many different Boolean functions of degree 4 are there ?

(A) 2^4
(B) 2^8
(C) 2^{12}
(D) 2^{16}

UGCNET2013-II(JUN)

16. Let f and g be the functions from the set of integers to the set integers defined by
f(x) = 2x + 3 and g(x) = 3x + 2
Then the composition of f and g and g and f is given as
(A) 6x + 7, 6x + 11
(B) 6x + 11, 6x + 7
(C) 5x + 5, 5x + 5
(D) None of the above

UGCNET2013-II(Dec.)

17. Consider a set A = {1, 2, 3, .., 1000}. How many members of A shall be divisible by 3 or by 5 or by both 3 and 5 ?
(A) 533
(B) 599
(C) 467
(D) 66

UGCNET2014-II(DEC)

18. If we define the functions f, g and h that map R into R by : f(x) = x^4, g(x) = $\sqrt{x^2+1}$, h(x) = x^2 + 72, then the value of the composite functions ho(gof) and (hog)of are given as
(A) x^8 - 71 and x^8 - 71
(B) x^8 - 73 and x^8 - 73
(C) x^8 + 71 and x^8 + 71
(D) x^8 + 73 and x^8 + 73

UGCNET2014-II(DEC)

19. Let A and B be sets in a finite universal set U. Given the following : $|A-B|, |A \bigoplus B|, |A| + |B|$ and $|A \cup B|$ Which of the following is in order of increasing size ?
(A) $|A-B| < |A \bigoplus B| < |A| + |B| < |A \cup B|$
(B) $|A \bigoplus B| < |A-B| < |A \cup B| < |A| + |B|$
(C) $|A \bigoplus B| < |A| + |B| < |A-B| < |A \cup B|$
(D) $|A-B| < |A \bigoplus B| < |A \cup B| < |A| + |B|$

UGCNET2016-II(aug.)

20. How many different equivalence relations with exactly three different equivalence classes are there on a set with five elements?
(A) 10
(B) 15
(C) 25
(D) 30

UGCNET2016-II(jun.)

21. Suppose that R_1 and R_2 are reflexive relations on a set A. Which of the following statements is correct?
(A) $R_1 \cap R_2$ is reflexive and $R_1 \cup R_2$ is irreflexive.
(B) $R_1 \cap R_2$ is irreflexive and $R_1 \cup R_2$ is reflexive.
(C) Both $R_1 \cap R_2$ and $R_1 \cup R_2$ are reflexive.
(D) Both $R_1 \cap R_2$and $R_1 \cup R_2$ are irreflexive.

UGCNET2016-II(jun.)

22. Which of the following logic expressions is incorrect?
(A) $1 \oplus 0 = 1$
(B) $1 \oplus 1 \oplus 1 = 1$
(C) $1 \oplus 1 \oplus 0 = 1$
(D) $1 \oplus 1 = 1$

UGCNET2016-II(jun.)

23. In how many ways can the string
$A \cap B - A \cap B - A$
be fully parenthesized to yield an infix expression?
(A) 15
(B) 14
(C) 13
(D) 12

UGCNET2016-II(jun.)

24. The set of all Equivalence Classes of a set A of Cardinality C
(A) is of cardinality 2^c
(B) have the same cardinality as A
(C) forms a partition of A
(D) is of cardinality C^2

ISRO 2007

25. Which one of the following is true?
(A) $R \cap S = (R \cup S) - [(R-S) \cup (S-R)]$
(B) $R \cup S = (R \cap S) - [(R-S) \cup (S-R)]$
(C) $R \cap S = (R \cup S) - [(R-S) \cap (S-R)]$
(D) $R \cap S = (R \cup S) \cup (R-S)$

ISRO 2011

26. The number of elements in the power set of the set A, B, C is
(A) 7
(B) 8
(C) 3
(D) 4

ISRO 2013

27. Let A be a finite set having x elements and let B be a finite set having y elements. What is the number of distinct functions mapping B into A.
(A) x^y
(B) $2^{(x+y)}$
(C) y^x
(D) $y!/(y-x)!$

ISRO 2014

28. The symmetric difference of sets $A = \{1,2,3,4,5,6,7,8\}$ and $B = \{1,3,5,6,7,8,9\}$ is:
(A) $\{1,3,5,6,7,8\}$
(B) $\{2,4,9\}$
(C) $\{2,4\}$
(D) $\{1,2,3,4,5,6,7,8,9\}$
ISRO 2017

29. Let S be an infinite set and $S_1 \ldots, S_n$ be sets such that $S_1 \cup S_2 \cup \cdots \cup S_n = S$. Then
(A)at least one of the set S_i is a finite set
(B)not more than one of the set S_i can be finite
(C)at least one of the sets S_i is an infinite
(D)not more than one of the sets S_i can be infinite
(E)None of the above
GATE 1993,2 MARKS

30. Let A be a finite set of size n. The number of elements in the power set of $A \times A$ is:
(A)2^{2^n}
(B)2^{n^2}
(C)$(2^2)^2$
(D)$(2^2)^n$
(E)None of the above
GATE 1993,2 MARKS

31. Let A and B be sets with cardinalities m and n respectively. The number of one-one mappings from A to B, when $m < n$, is
(A)m^n
(B)^nP_m
(C)^mC_n
(D)^nC_m
(E)^mP_n

GATE 1993,1 MARK

32. Let R be a symmetric and transitive relation on a set A. Then
(A)R is reflexive and hence an equivalence relation
(B)R is reflexive and hence a partial order
(C)R is reflexive and hence not an equivalence relation
(D)None of the above
GATE 1995,1 MARK

33. The number of elements in the power set $P(S)$ of the set $S = \{\{\phi\}, 1, \{2,3\}\}$ is:
(A)2
(B)4
(C)8
(D)None of the above
GATE 1995,1 MARK

34. Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. The set $(A-B) \cup (B-A) \cup (A \cap B)$ is equal to
(A)$A \cup B$
(B)$A^c \cup B^c$
(C)$A \cap B$
(D)$A^c \cap B^c$
GATE 1996,2 MARKS

35. Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this one can conclude that
(A)$|X| = 1, |Y| = 97$
(B)$|X| = 97, |Y| = 1$
(C)$|X| = 97, |Y| = 97$
(D)None of the above
GATE 1996,2 MARKS

36. The number of equivalence relations of the set $\{1,2,3,4\}$ is
(A) 15
(B) 16
(C) 24
(D) 4
GATE 1997,2 MARKS

37. Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is
(A) n
(B) n^2
(C) 1
(D) $n+1$
GATE 1998,1 MARK

38. Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:
(i)$R_1 \cup R_2$ is an equivalence relation
(ii)$R_1 \cap R_2$ is an equivalence relation
Which of the following is correct?
(A) Both assertions are true
(B) Assertions (i) is true but assertions (ii) is not true
(C) Assertions (ii) is true but assertions (i) is not true
(D) Neither (i) nor (ii) is true
GATE 1998,1 MARK

39. The number of functions from an m element set to an n element set is
(A) $m+n$
(B) m^n
(C) n^m
(D) m^*n

GATE 1998,1 MARK

40. The binary relation $R = \{(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)\}$ on the set $A = \{1, 2, 3, 4\}$ is
(A) reflective, symmetric and transitive
(B) neither reflective, nor irreflexive but transitive
(C) irreflexive, symmetric and transitive
(D) irreflexive and antisymmetric
GATE 1998,2 MARKS

41. In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada whereas 13 persons speak both Kannada and English. How many people speak all three languages?
(A) 9
(B) 8
(C) 7
(D) 6
GATE 1998,2 MARKS

42. The number of binary relations on a set with n elements is:
(A) n^2
(B) 2^n
(C) 2^{n^2}
(D) None of the above
GATE 1999,1 MARK

43. A relation R is defined on the set of integers as xRy iff $(x + y)$ is even. Which of the following statements is true?
(A) R is not an equivalence relation
(B) R is an equivalence relation having 1 equivalence class
(C) R is an equivalence relation having 2 equivalence classes
(D) R is an equivalence relation having 3 equivalence classes
GATE 2000,2 MARKS

44. Let P(S) denotes the powerset of set S. Which of the following is always true?
(A) P(P(S))=P(S)
(B) P(S)$\cap P$(P(S)) = $\{\phi\}$
(C) P(S) $\cap$S = P(S)
(D) S $\notin P$(S)
GATE 2000,2 MARKS

45. Consider the following relations:
R1 (a, b) iff $(a + b)$ is even over the set of integers
R2 (a, b) iff $(a + b)$ is odd over the set of integers
R3 (a, b) iff $a.b > 0$ over the set of non-zero rational numbers
R4 (a, b) iff $|a - b| \leq 2$ over the set of natural numbers
Which of the following statements is correct?
(A) R1 and R2 are equivalence relations, R3 and R4 are not
(B) R1 and R3 are equivalence relations, R2 and R4 are not
(C) R1 and R4 are equivalence relations, R2 and R3 are not
(D) R1, R2, R3 and R4 are all equivalence relations
GATE 2001,1 MARK

46. Consider the following statements:
S1: There exists infinite sets A, B, C such that A∪(B∪ C) is finite.
S2: There exists two irrational numbers x and y such that (x+y) is rational.
Which of the following is true about S1 and S2?
(A) Only S1 is correct
(B) Only S2 is correct
(C) Both S1 and S2 are correct
(D) None of S1 and S2 is correct
GATE 2001,2 MARKS

47. Let $f : A \to B$ a function, and let E and F be subsets of A. Consider the following statements about images.
$S1 : f(E \cup F) = f(E) \cup f(F)$
$S2 : f(E \cap F) = f(E) \cap f(F)$
Which of the following is true about S1 and S2?
(A) Only S1 is correct
(B) Only S2 is correct
(C) Both S1 and S2 are correct
(D) None of S1 and S2 is correct
GATE 2001,2 MARKS

48. The binary relation S = ϕ (empty set) on set A = {1,2,3} is
(A) Neither reflexive nor symmetric
(B) Symmetric and reflexive
(C) Transitive and reflexive
(D) Transitive and symmetric
GATE 2002,2 MARKS

49. Consider the binary relation:

S={(x,y) |y=x+1 and x,y∈{0,1,2}}

The reflexive transitive closure of S is
(A){(x,y) $|y > x$ and x,y∈{0,1,2}}
(B){(x,y) $|y \geq x$ and x,y∈{0,1,2}}
(C){(x,y) $|y < x$ and x,y∈{0,1,2}}
(D){(x,y) $|y \leq x$ and x,y∈{0,1,2}}
GATE 2004,1 MARK

50. The number of different n × n symmetric matrices with each element being either 0 or 1 is : (Note: power (2,x) is same as 2^x)
(A) power (2,n)
(B) power(2,n^2)
(C) power (2, (n^2+n/2)
(D) power (2, (n^2– n)/2)
GATE 2004,1 MARK

51. Let A, B and C be non-empty sets and let
X = (A - B) - C and Y = (A - C) - (B - C)
Which one of the following is TRUE?
(A) X = Y
(B) X ⊂ Y
(C)Y ⊂ X
(D) None of these
GATE 2005,1 MARK

52. Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE?
(A) R ∪ S, R ∩ S are both equivalence relations.
(B) R ∪ S is an equivalence relation.
(C) R ∩ S is an equivalence relation.
(D) Neither R ∪S nor R ∩ S is an equivalence relation
GATE 2005,2 MARK

53. Let f: B → C and g: A→ B be two functions let h = (f o g).Given that h is an onto function which one of the following is TRUE?
(A) f and g should both be onto functions
(B) f should be onto but g need to be onto
(C) g should be onto but f need not be onto
(D) both f and g need to be onto
GATE 2005,2 MARK

54. Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are:
(A) n and n
(B) $n^2$and n
(C) n^2 and 0
(D) n and 1
GATE 2007,1 MARK

55. What is the maximum number of different Boolean functions involving n Boolean variables?
(A) n^2
(B) 2^n
(C) 2^{2^n}
(D) 2^{n^2}
GATE 2007,1 MARK

56. If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup (Q^c \cup R^c)$
(A) $Q^c \cup R^c$
(B) $P \cup Q^c \cup R^c$
(C) $P^c \cup Q^c \cup R^c$
(D) U
GATE 2008,1 MARK

57. Consider the binary relation R = {(x,y), (x,z), (z,x), (z,y)} on the set {x,y,z} . Which one of the following is TRUE?
(A) R is symmetric but NOT antisymmetric
(B) R is NOT symmetric but antisymmetric
(C) R is both symmetric and antisymmetric
(D) R is neither symmetric nor antisymmetric
GATE 2009,1 MARK

58. What is the possible number of reflexive relations on a set of 5 elements ?
(A) 2^{10}
(B) 2^{15}
(C) 2^{20}
(D) 2^{25}
GATE 2010,1 MARK

59. How many onto (or surjective) functions are there from an n-element (n ≥ 2) set to a 2-element set?
(A) 2^n
(B) 2^n-1
(C) 2^n-2
(D) $2(2^n - 2)$
GATE 2012,2 MARKS

60. Let ***S*** denote the set of all functions ***f***:$\{0,1\}^4$ → {0,1} . Denote by ***N*** the number of functions from ***S*** to the set {0,1} .The value of $\log_2 \log_2$ N is______
GATE 2014-I,2 MARKS

61. A non-zero polynomial $f(x)$ of degree 3 has roots at$x = 1, x = 2$ and $x = 3$. Which one of the following must be TRUE?
(A) $f(0)f(4) < 0$
(B) $f(0)f(4) > 0$
(C) $f(0) + f(4) > 0$
(D) $f(0) + (4) < 0$
GATE 2014-II,1 MARK

62. Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets and of we say U < V if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements:
S1: There is a subset of that is larger than every other subset.
S2: There is a subset of that is smaller than every other subset.

Which one of the following is CORRECT?
(A) Both S1 and S2 are true
(B) S1 is true and S2 is false
(C) S2 is true and S1 is false
(D) Neither S1 nor S2 is true

GATE 2014-II,2 MARKS

63. For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True ?
1.$\Phi \varepsilon\ 2^A$
2.$\Phi \subseteq 2^A$
3.{5,{6}}$\varepsilon\ 2^A$
4.{5,{6}}$\subseteq 2^A$
(A) 1 and 3 only
(B) 2 and 3 only
(C) 1, 2 and 3 only
(D) 1, 2 and 4 only

GATE 2015-I,1 MARK

64. If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$ then $\frac{g(h(x))}{h(g(x))}$ is :
(A)$\frac{h(x)}{g(x)}$
(B)$\frac{-1}{x}$
(C)$\frac{g(x)}{h(x)}$
(D)$\frac{x}{(1-x)^2}$

GATE 2015-I,1 MARK

65. The number of divisors of 2100 is ________
GATE 2015-II,1 MARK

66. Let R be the relation on the set of positive integers such that aRb and only if a and b are distinct and let have a common divisor other than 1. Which one of the following statements about R is true?
(A) R is symmetric and reflexive but not transitive
(B) R is reflexive but not symmetric not transitive
(C) R is transitive but not reflexive and not symmetric
(D) R is symmetric but not reflexive and not transitive
GATE 2015-II,1 MARK

67. The cardinality of the power set of $\{0, 1, 2, \ldots, 10\}$ is________
GATE 2015-II,1 MARK

68. The number of onto functions (surjective functions) from set $X = \{1, 2, 3, 4\}$ to set $Y = \{a, b, c\}$ is _______

GATE 2015-II,2 MARKS

69. Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y. Let f be randomly chosen from F. The probability of f being one-to-one is .....
GATE 2015-II,2 MARKS

70. Suppose U is the power set of the set S={1,2,3,4,5,6}.For any T ∈ U ,let $|T|$ denote the number of element in T and T' denote the complement of T. For any T,R ∈ U ,let T\R be the set of all elements in T which are not in R.Which of the following is true?
(A)$\forall X \in U(|X| = |X'|)$
(B)$\exists X \in U \exists Y \in U(|X| = 5, |Y| = 5\ and\ X \cap Y = \phi)$
(C)$\forall X \in U \forall Y \in U(|X| = 2, |Y| = 3\ and\ X \setminus Y = \phi)$
(D)$\forall X \in U \forall Y \in U(X \setminus Y = Y' \setminus X')$
GATE 2015-III,1 MARK

71. Let R be a relation on the set of ordered pairs of positive integers such that ((p,q),(r,s))∈ R if and only if p - s = q - r. Which one of the following is true about R ?
(A) Both reflexive and symmetric
(B) Reflexive but not symmetric
(C) Not reflexive but symmetric
(D) Neither reflexive nor symmetric
GATE 2015-III,2 MARKS

72. A function f : $N^+ \to N^+$, defined on the set of positive integers N^+, satisfies the following properties:

f (n) = f (n / 2) if n is even
f (n) = f (n + 5) if n is odd

Let R = $\{i\ |\exists\ j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is ______________ .
GATE 2016-I,2 MARKS

73. Let f(x) be a polynomial and g(x) = f′(x) be its derivative. If the degree of (f(x)+ f(-x)) is10, then the degree of (g(x)-g(-x)) is______________.
GATE 2016-II,1 MARKS

74. A binary relation R on N× N is defined as follows: (a, b)R(c, d) if a ≤ c or b ≤ d. Consider the following propositions:
P: R is reflexive
Q: R is transitive
Which one of the following statements is TRUE?

(A)Both P and Q are true.
(B)P is true and Q is false.
(C)P is false and Q is true.
(D)Both P and Q are false.

GATE 2016-II,2 MARKS

75. Consider a set U of 23 different compounds in a Chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U . Consider the following statements:
   I. Each compound in U \ S reacts with an odd number of compounds.
   II. At least one compound in U \ S reacts with an odd number of compounds.
   III. Each compound in U \ S reacts with an even number of compounds.

   Which one of the above statements is ALWAYS TRUE?
   (A)Only I
   (B)Only II
   (C)Only III
   (D)None

GATE 2016-II,2 MARKS

76. The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is___________

GATE 2017-I,2 MARKS

77. Let N be the set of natural numbers. Consider the following sets,
   P: Set of Rational numbers (positive and negative)
   Q: Set of functions from 0,1 to N
   R: Set of functions from N to 0, 1
   S: Set of finite subsets of N
   Which of the above sets are countable?
   (A) Q and S only
   (B) P and S only
   (C) P and R only
   (D) P, Q and S only

GATE 2018,2 MARKS

Solutions

1. Ans:c
Idempotent Rule :
$A \vee A = A$
$A \wedge A = A$.

2. Ans:b
$f(x) = x + 1$
$f(f(x)) = f(x+1) = x + 2$
$f(f(f(x))) = f(x+2) = x + 3$
$f(f(f(f(x)))) = f(x+3) = x + 4 = g(x) + 1$.

3. Ans:c
As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) \leq P(A) + P(B)$, Hence Option c is always true.
Option A is true only if events are independent.Option B is true when events are mutually exclusive.

4. Ans:c
R=$\{(a,b),(b,c),(c,a)\}$ The transitive closer would be: $(a,b),(b,c) \rightarrow (a,c)$
$(a,c),(c,a) \rightarrow (a,a)$
$(b,c),(c,a) \rightarrow (b,a)$
$(b,a),(a,b) \rightarrow (b,b)$
$(c,a),(a,b) \rightarrow (c,b)$
$(c,b),(b,c) \rightarrow (c,c)$.

5. Ans:a
Domain of A and B lies between -1 and 1 . So for every element in A there is unique element in B . Hence it is injective . There are some elements in B , which are not the part of mapping ,there must be 2 ∈A to map 1 ∈B but 2 is not in the domain of A , Hence it is not surjective.

6. Ans:b
Idempotent Rule :
$A \vee A = A$
$A \wedge A = A$.

7. Ans:d
A:False, (1,1) is not present. Hence not reflexive.
B:False , Corresponding to (1,2) ordered pair , (2,1) is not present , hence not symmetric.
C:False, (1,2) and (2,3) are present but (1,3) is not present , hence not transitive.

8. Ans:d
Transitive relation :
${}_AR_B$ & ${}_BR_C$ then ${}_AR_C$.
Given problem statements corresponds to transitive relation.

9. Ans:b
Number between 1 and 250 divisible by 2,5 and 7 are $\lfloor \frac{250}{70} \rfloor$=3
[$\because$ LCM of 2,5, and 7 is 70].

10. Ans:c
For all positive values of x, $\sqrt{2x+3}$ will be positive and y will be negative. So $x \geq 0$, $y < 0$, Nearest possible answer is option c.

11. Ans:d
Symmetric:
Lets assume relation is represented by a matrix, for symmetric relation $A[i][j] = A[j][i]$, For example if $A[1][3] = 1$, then $A[3][1] = 1$ and if $A[2][4] = 0$, then $A[4][2] = 0$.Which imply lower triangular is bound/-fixed and depend on the elements of upper triangular. Every element on the upper triangular is free and have 2 choices {0,1}. Similarly , diagonal elements are also free and have 2 choices for each element.
Number of diagonal elements $=n$
Number of upper triangular elements $= (n-1)+(n-2)+\cdots+1 = n\frac{(n-1)}{2} = \frac{(n^2-n)}{2}$
Number of free elements $=\frac{(n^2-n)}{2} + n = \frac{(n^2+n)}{2}$
Total number of symmetric relations= $2^{\frac{(n^2+n)}{2}}$ [$\because$ Every element have 2 choices]
Reflexive :In the same way we can find the number of reflexive and symmetric relations, here diagonal elements will be fixed to 1 . Hence number of free elements are $=\frac{(n^2-n)}{2}$
Number of reflexive and symmetric relations are $=2^{\frac{(n^2-n)}{2}}$.

12. Ans:b
Number of elements in the power set P(A) of a set(A) having n elements are 2^n
Here n=1, so P(A)=$2^1 = 2$
$A = \{\phi\}, P(A) = \{\phi, \{\phi\}\}$.

13. Ans:b
A = {2, 3, 5, 7} and B= {2, 5, 8, 9}, A∪

B={2,3,5,7,8,9}
$|A \cup B|$=6, $|P(A \cup B)| = 2^6 = 64$.

14. Ans:b
$\forall x \in Z^+, (x, x)$ is possible x/x is always true, hence it is reflexive .
$\exists_{x,y}, (x, y) \in R \not\leftrightarrow (y, x) \in R$ for example ordered pair (3,6) is possible (6/3) but (6,3) is not possible.
Hence given relation R is not symmetric but antisymmetric due to (x, x) pair.
Suppose x divides y, and y divides z. Then $\exists_{a,b}$, so that $y = a \times x$ and $z = b \times y = b \times a \times x$
Hence given relation is transitive .

15. Ans:d
Number of boolean functions possible(N) with n variables are $=2^{2^n}$
Here, $n = 4, N = 2^{2^4} = 2^{16}$.

16. Ans:a
$f \circ g(x) = f(g(x)) = f(3x+2) = 2(3x+2)+3 = 6x+7$
$g \circ f(x) = g(f(x)) = g(2x+3) = 3(2x+3)+2 = 6x+11$
.

17. Ans:c
$N(A)$ Number is divisible by 3: $\lfloor 1000/3 \rfloor = 333$
$N(B)$ Number is divisible by 5:$\lfloor 1000/5 \rfloor = 200$
$N(A \cap B)$Number is divisible by 3 and 5:$\lfloor 1000/15 \rfloor =$ 66
$N(A \cup B)$ Number is divisible by 3 or 5 or both.
$N(A \cup B) = N(A) + N(B) + N(A \cap B)$
=333+200-66=467

18. Ans:d
$f(x) = x^4$
$g(x) = \sqrt{x+1}$
$h(x) = x^2 + 72$
$g \circ f(x) = g(x^4) = \sqrt{x^4+1}$
$h \circ g \circ f(x) = h(\sqrt{x^4+1}) = x^4 + 1 + 72$
$= x^4 + 73$.

19. Ans:d
$A - B = A \cap B^c$
$A \oplus B = (A \cap B^c) \cup (A^c \cap B)$
Clearly $|A - B| < |A \oplus B|$
Also, $|A \oplus B|$ include all the elements of A and B , excepts the common elements.
$|A \cup B|$ include all the elements of A and B , and common elements are included only once .
$|A| + |B|$: includes all elements of A and B and common elements are counted twice.
Hence option d is correct.

20. Ans:c
Recurrence relation for k equivalence classes on the set of n elements is given as:
$p(n, k) = p(n-1, k-1) + k * p(n-1, k), p(x, x) = p(x, 1) = 1$
$p(5, 3) = p(4, 2) + 3 * p(4, 3) = 7 + 3 * 6 = 25$
$p(4, 2) = p(3, 1) + 2 * p(3, 2) \rightarrow 1 + 2 * 3 = 7$
$p(3, 2) = p(2, 1) + 2 * p(2, 2) \rightarrow 1 + 2 * 1 = 3$
$p(4, 3) = p(3, 2) + 3 * p(3, 3) = 3 + 3 * 1 = 6$.

21. Ans:c
A={1,2,3}
$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3)\}$
$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 3)\}$
$R_1 \cap R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$
$R_1 \cup R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (2, 3)\}$
Intersection and Union of two reflexive sets are always reflexive.

22. Ans:c,d
$a \oplus b = a'b + ab'$
A:$1 \oplus 0 = 1'0 + 10' = 1$, Hence correct
B:$1 \oplus 1 \oplus 1 = 1 \oplus (1 \oplus 1) = 1 \oplus 0 = 1$, Hence correct.
C:$1 \oplus 1 \oplus 0 = 1 \oplus (1 \oplus 0) = 1 \oplus 1 = 0$, Hence Incorrect.
D:$1 \oplus 1 = 0$, Hence Incorrect.

23. Ans:b
C_n is the number of different ways $n + 1$ factors can be completely parenthesized
$C((2n), n)/(n + 1) = C(10, 5)/6 = 14$.

24. Ans:c
.

25. Ans:a
Symmetric difference - suppose A and B are 2 sets then symmetric difference of A and B is $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$. So, $(R - S) \cup (S - R) = (R \cup S) - (R \cap S)$ A)$(R \cup S) - ((R \cup S) - (R \cap S))$ $\equiv (R \cap S)$ ≡LHS.

26. Ans:d
Number of elements in the power set of a set having n elements are 2^n
Here n=2, Number of elements in the power set= $2^2 = 4$.

27. Ans:a
Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. Then number of functions from X to Y will be $|Y|^{|X|}$ and number of functions from Y to X will be $|X|^{|Y|}$
Here, $|A| = x, |B| = y$, Number of functions from B to A are x^y.

28. Ans:b
Symmetric difference ($\triangle$)- suppose A and B are 2 sets then symmetric difference of A and B is $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.
A= {1,2, 3, 4, 5, 6, 7, 8} and B = { 1,3, 5, 6, 7, 8, 9}
$A \triangle B$={2,4,9}.

29. Ans:c
Union of given sets $S_1, S_2, ...S_n$ will be infinite when any of these set is infinite . Also , if all the sets are infinite then their union will infinite.
A:False, If all the sets are infinite then their union will also be infinite.
B:False,If $S_1, S_2, ...S_{n-1}$ are finite and S_n is infinite then also S will be infinite.
C:True .

D:False, if all the sets $S_1, S_2, ...S_n$ are infinite then S will be infinite.

30. Ans:b
Number of elements in the power set of n elements will be 2^n. Let $A = \{1, 2, 3\}$
$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
$|A| = 3, |A \times A| = 9$
By induction,
$|A| = n, |A \times A| = n^2$
Number of elements in the power set of n^2 elements will be 2^{n^2}.

31. Ans:b
Let $A = a_1, a_2.....a_m$ and $B = b_1......B_n$. A one-to-one function f assigns each element a_i of A a distinct element $b_j = f(a_i)$ of B; for a_1 there are n choices, for a_2 there are $n-1$ choices.
Number of choices $=n \times (n-1)......(n-(m-1) = {}^nP_m$.

32. Ans:d
Let A={a,b,c} and relation R={(b,c),(c,b),(b,b),(c,c)}. R is symmetric and transitive but not reflexive.

33. Ans:c
$P(S) = \{\phi, \{\{\phi\}\}, \{1\}, \{\{2, 3\}\}, \{\{\phi\}, 1\}, \{1, \{2, 3\}\}, \{\{\phi\}, \{2, 3\}\}, \{\{\phi\}, 1, \{2, 3\}\}\}$
$|P(S)| = 8$.

34. Ans:a
Given statement can be solved using boolean algebra , $A - B \equiv A \cap B'$
$(A - B) \cup (B - A) \cup (A \cap B)$
$A \cap B' + B \cap A' + A \cap B$
$A \cup B$.

35. Ans:a
Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. Then number of functions will be $|Y|^{|X|}$
$|Y|^{|X|}$=97
$|Y| = 97, |X| = 1$.

36. Ans:a
Corresponding to any partition of X, there exists an equivalence relation $\sim$ on X.So we can calculate number of equivalence relation by finding the number of partitions. The total number of partitions of an n-element set is the Bell number
$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k}$$
Where $B_0 = B_1 = 1$
$B_2 = \binom{1}{0} * B_0 + \binom{1}{1} * B_1 = 1 + 1 = 2$
$B_3 = \binom{2}{0} * B_0 + \binom{2}{1} * B_1 + \binom{2}{2} * B_2 = 1 + 2 + 2 = 5$
$B_4 = \binom{3}{0} * B_0 + \binom{3}{1} * B_1 + \binom{3}{2} * B_2 + \binom{3}{3} * B_3 = 1 + 3 + 6 + 5 = 15$.

37. Ans:b
Equivalence relation R on A can be defined as $R : A \to A$ if R is reflexive , transitive and symmetric.
Let $A = \{1, 2, 3\}$
Smallest Equivalence relation $R : A \to A$ ={(1,1),(2,2),(3,3)}
Largest Equivalence relation $R : A \to A$ = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}
So, size of smallest equivalence relation is n and largest equivalence relation is n^2.

38. Ans:c
Lets take an example to understand this :
Let R_1 and R_2 are defined on set S={1,2,3}
R_1={(1,1),(2,2),(3,3),(1,3),(3,1)}
R_2={(1,1),(2,2),(3,3),(3,2),(2,3)}
$R_1 \cap R_2$={(1,1),(2,2),(3,3)}
which is equivalence relation
$R_1 \cup R_2$
={(1,1),(2,2),(3,3),(1,3),(3,1),(3,2),(2,3)}
$R_1 \cup R_2$ is not equivalence relation because it is not transitive relation.

39. Ans:c
The number of functions from an m element set to an n element set is n^m.

40. Ans:b
Given relation is not reflective because ordered pair (4,4) is not present.
Given relation is not irreflective because ordered pair (1,1), (2,2), (3,3) should not be there.
Given relation is reflexive.

41. Ans:d
$N(A \cup B \cup C) = 28, N(A \cap B) = 9, N(B \cap C) = 11, N(A \cap C) = 13, N(A) = 18, N(B) = 15, N(C) = 22$
$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C)$
$28 = 18 + 15 + 22 - [9 + 11 + 13] + N(A \cap B \cap C)$
$N(A \cap B \cap C) = 6$

42. Ans:c
Any Relation R is defined as $R \subseteq A \times A$, where set A contains n elements . $|A \times A| = n^2$. Number of relations will be subset of $|A \times A|$.Number of subsets of a set having n^2 elements are 2^{n^2}.

43. Ans:c
Relation R is always reflexive as $\forall x(x, x)$ is even.
Relation R is symmetric , if $(x+y)$ is even then $(y+x)$ will also be even.
Relation R is transitive ,if $(x+y)$ and $(y+z)$ are even then $(x+z)$ will also be even.
$x + y$ are even iff:
1. Both x and y are even.
2. Both x and y are odd.
So relation R is equivalence relation having 2 equivalence classes.

44. Ans:b
A:false , because a set can not be equal to powerset.
B:True .
C: False , As $P(S) \cap S = \phi$,
D:False , every set belong to its powerset .

45. Ans:b
1.Relation R1 is always reflexive as $\forall x(x+x)$ is even.
Relation R1 is symmetric , if $(x+y)$ is even then $(y+x)$ will also be even.
Relation R1 is transitive ,if $(x+y)$ and $(y+z)$ are even then $(x+z)$ will also be even.
Hence relation R1 is equivalence relation.
2.Relation R2 is not reflexive as $\forall x(x,x)$ is always even, hence not equivalence relation.
3.Relation R3 is always reflexive as $\forall x(x.x) > 0$.
Relation R3 is symmetric , if $(x.y) > 0$ is even then $(y.x) > 0$.
Relation R3 is transitive ,if $(x.y) > 0$ and $(y.z) > 0$ then $(x.z) > 0$
4.Relation R4 is not transitive $|a-b| <= 2$ and $|b-c| <= 2$, doesn't mean $|a-c| <= 2$.
So relation R1 and R3 will be equivalence relation.

46. Ans:c
S_1: is correct, as A is infinite and $B \cup C$ is also infinite , but $A \cap (B \cup C)$ could be empty set (there is no common elements between A and $B \cup C$). Empty set is finite.
S_2: Correct, Let $A = 5+\sqrt{5}, B = 2-\sqrt{5}, A+B = 7$, which is rational.

47. Ans:a
$S_1 : f(E \cup F) = f(E) \cup f(F)$
$S_2 : f(E \cap F) <= f(E) \cap f(F)$
If f is injective then S_2 is also true , but for any arbitrary function , it is not true. .

48. Ans:d
Empty relation ϕ is symmetric and transitive but not reflexive. For relation to be reflexive there should be ordered pair {(1,1),(2,2),(3,3)}.

49. Ans:b
Given relation R = { (0,1) , (1,2) }
Reflexive closure ={ (0,1) (1,2) (0,2) (0,0) (1,1) (2,2) }
Transitive closure = { (0,1) (1,2) (0,2) }
Reflexive Transitive closure ={ (0,1) (1,2) (0,2) (0,0) (1,1) (2,2) }.

50. Ans:c
For symmetric relation $A[i][j] = A[j][i]$, For example if $A[1][3] = 1$, then $A[3][1] = 1$ and if $A[2][4] = 0$, then $A[4][2] = 0$.Which imply lower triangular is bound and depend on the elements of upper triangular. Every element on the upper triangular is free and have 2 choices {0,1}. Similarly , diagonal elements are also free and have 2 choices for each element.
Number of diagonal elements $=n$
Number of upper triangular elements $= (n-1)+(n-2)+\cdots+1 = n\frac{(n-1)}{2} = \frac{(n^2-n)}{2}$
Number of free elements $= \frac{(n^2-n)}{2} + n = \frac{(n^2+n)}{2}$
Total number of symmetric relations= $2^{\frac{(n^2+n)}{2}}$ [$\because$ Every element have 2 choices] .

51. Ans:a
Given statement can be solved using boolean algebra , $A - B \equiv A \cap B'$
$X = (A-B) - C$
$\equiv (A \cap B') \cap C'$
$\equiv A \cap B' \cap C'$
$Y = (A-C) - (B-C)$
$\equiv (A \cap C') - (B \cap C')$
$\equiv (A \cap C') \cap (B \cap C')'$
$\equiv (A \cap C') \cap (B' \cup C)$
$\equiv (A \cap C' \cap B') \cup (A \cap C' \cap C)$
$\equiv (A \cap C' \cap B')$ $X = Y$.

52. Ans:c
Lets take an example to understand this :
Let R and S are defined on set A={1,2,3}
R={(1,1),(2,2),(3,3),(1,3),(3,1)}
S={(1,1),(2,2),(3,3),(3,2),(2,3)}
$R \cap S$={(1,1),(2,2),(3,3)}
which is equivalence relation
$R \cup S$
={(1,1),(2,2),(3,3),(1,3),(3,1),(3,2),(2,3)}
$R \cup S$ is not equivalence relation because it is not transitive relation .

53. Ans:b
$A = \{a,b,c\}, B = \{a,d,e,g\}, C = \{d,e,f\}$
$f = \{(a,d),(d,f),(e,e),(cf,d)\}$
$g = \{(b,d),(c,e),(a,a)\}$
$h = \{(a,d),(b,f),(c,e)\}$
Now, f and h must be onto function but g is not required to be onto.

54. Ans:b
Equivalence relation R on A can be defined as
$R : A \to A$ if R is reflexive , transitive and symmetric.
Let $A = \{1,2,3\}$
Smallest Equivalence relation $R : A \to A$
={(1,1),(2,2),(3,3)}
Largest Equivalence relation $R : A \to A$
= {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2), (3,3)}
So, size of smallest equivalence relation is n and largest equivalence relation is n^2.

55. Ans:c
Number of boolean functions possible(N) with n variables are $=2^{2^n}$.

56. Ans:d
$(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup (Q^c \cup R^c)$
$\equiv (Q \cap R) \cap (P \cup P^c) \cup (Q^c \cup R^c)$
$\equiv (Q \cap R) \cup (Q^c \cup R^c)$

$\equiv (Q \cap R) \cup (Q \cap R)^c$
$\equiv U.$

57. Ans:d
Given relation is not symmetric because ordered pair (y,x) , (y,z) is not present . Also R is not antisymmetric as ordered pair (x,z) is present corresponding to (z,x). Hence option d is correct.

58. Ans:c
A relation $R : A \to A$ is defined on set A having n elements then number of reflexive relations will be 2^{n^2-n}
Here $n = 5$, So number of reflexive relations are $2^{25-5} = 2^{20}$.

59. Ans:c
Number of onto functions from $|A| = m$ to $|B| = n$
$n^m - C(n,1)(n-1)^m + C(n,2)(n-2)^m - ... + (-1)^{n-1}C(n,n-1).1^m$
$2^n - 2 * 1^n$
$2^n - 2.$

60. Ans:16
$\{0,1\}^4$ denotes binary strings of length 4. There are 16 such strings are possible. Number of functions from a set having 16 numbers to a set having 2 numbers are 2^{16},i.e. $S = 2^{16}$
N denotes set of function from S to {0,1}, which is $2^{2^{16}}$, i.e. $N = 2^{2^{16}}$
$\log_2 \log_2 2^{2^{16}} = 16.$

61. Ans:a
The roots are $x = 1, x = 2, andx = 3.$
$f(x) = (x-1)(x-2)(x-3)$
$f(0) = -6, f(4) = 6$
$f(0)f(4) < 0.$

62. Ans:a
Symmetric difference(denoted by $\triangle$) of 2 sets A and B is= $(A \cup B) - (A \cap B)$, i.e symmetric difference include all the elements of A and B and exclude all common elements .
For example $A = \{1,2,6,7\}, B = \{2,3,4,5,6\}, A \triangle B = \{1,3,4,5,7\}$
Also, minimum element is from A hence $A < B$
To understand given problem , lets consider a set $U = \{2,3,4,5,6\}$ and two subsets of A and B of U.
I: $A = \{\phi\}$(smallest subset), $B = \{2,4\}$ (any subset of U)
$A \triangle B = B$, So always $B < A$ and A will be the larger than all the subset of U.
II: $A = \{2,3,4,5,6\}$(Largest subset), $B = \{2,3,4\}$(any subset of U)
$A \triangle B = \{5,6\}$, Smallest element will always be from A, hence A will be the smallest element.

63. Ans:c
$A = \{5, \{6\}, \{7\}\}$
$2^A = \{\phi, \{5\}, \{\{6\}\}, \{\{7\}\}, \{5,\{6\}\}, \{5,\{7\}\},$
$\{\{6\},\{7\}\}, \{5,\{6\},\{7\}\}\}$
$\phi \in 2^A, \{5,\{6\}\} \in 2^A$ Hence 1 and 3 are correct.
Also, ϕ is subset of every set, hence 2 is also correct.
$\{5,\{6\}\} \not\subseteq 2^A$, because $\{5,\{6\}\} \notin 2^{2^A}$.

64. Ans:a
$g(h(x)) = 1 - \frac{x}{x-1} = \frac{-1}{x-1}$
$h(g(x)) = \frac{1-x}{-x}$
$\frac{g(h(x))}{h(g(x))} = \frac{x}{(1-x)(x-1)} = \frac{h(x)}{g(x)}$.

65. Ans:36
$2100 = 7 \times 3 \times 2^2 \times 5^2$
Total number of factors $= (1+1) \times (1+1) \times (2+1) \times (2+1)$
$= 2 \times 2 \times 3 \times 3$
$= 36.$

66. Ans:d
"aRb and only if a and b are distinct", So $\forall x \in Z^+, (x,x)$ ordered pair are not possible, hence R is not reflexive.
let consider some of the ordered pair in R: $\{3,6\}, \{6,8\}$, So {6,3} {8,6} always belong to R but {6,8}∉R, Hence R is symmetric but not reflexive and transitive.

67. Ans:2048
Cardinality of the given set=11.
Cardinality of power set of given set $=2^{11} = 2048$.

68. Ans:36
Number of onto functions from $|A| = m$ to $|B| = n$
$n^m - C(n,1)(n-1)^m + C(n,2)(n-2)^m - ... + (-1)^{n-1}C(n,n-1).1^m$
$3^4 -^3 C_1(3-1)^4 +^3 C_2(3-2)^4 -^3 C_3(3-3)^4$
$= 81 - 3 * 16 + 3 * 1 - 1 * 0$
$= 36$

69. Ans:.95
Number of One-to-one functions from a set having m elements to a set having n elements are ${}^nP_m$
Here $n = 20, m = 2$ number of one-one functions $=$ ${}^20P_2$
Probability = number of one-to-one functions / total number of functions = 20*19/20*20 = 0.95.

70. Ans:d
A: Incorrect, Let $X = \{1,2,3,5\}$,So X'={4,6}, $|X| = 4and|X'| = 2$
B: Incorrect, Let $X = \{1,2,3,4,5\}$,$Y = \{2,3,4,5,6\}$ $X \cap Y$={2,3,4,5}, For all possible set X and Y of length 5, $|X \cap Y| = 4$.
C: Incorrect, consider $X = \{1,2\}$, $Y = \{3,4,5\}$ and $X \setminus Y = \{1,2\}$ which is not null.
D: Correct, $X \ Y = X - Y$
$Y' \setminus X' = Y' - X' = (U - Y) - (U - X) = X - Y.$

71. Ans:c
If $((a,b),(a,b)) \in R$ then relation is reflexive
$((a,b),(a,b)) \leftrightarrow (a - b = b - a)$ Not correct, hence not reflexive
Symmetric:

$((a,b),(c,d)) \in R$ then $((c,d),(a,b)) \in R$
LHS:
$((a,b),(c,d)) \in R$ iff$(a-d=b-c)$
LHS:
$((c,d),(a,b)) \in R$ iff$(d-a=c-b)$
$(c-b=d-a) \equiv (d-a=c-b) \equiv (-(a-d)=-(b-c)) \equiv (a-d=b-c)$
Hence symmetric .

72. Ans:2
f(1) =f(6)=f(3)=f(8)=f(4)=f(2)=f(1) then repeat again. Here, we can analyze
f(1)=f(2)=f(3)=f(4)=f(6)=f(8)=A
Lets check other missing values.
f(5)=f(10)=f(5) then repeat.Hence
f(5)=f(10)=B
f(7)=f(12)=f(6)=A
f(9)=f(14)=f(7)=A
f(13)=f(18)=f(9)=A
f(17)=f(22)=f(11)=A
Hence given function takes 2 different values A and B.

73. Ans:9
$f(x) = x^{10}$,$g(x) = 10 \times x^9$
$f(-x) = x^{10}$,$g(-x) = -10 \times x^9$
$g(x) - g(-x) = 10.x^9 - \{-10x^9\} = 20.x^9$ Hence correct answer is 9.

74. Ans:b
It will be reflexive because ${}_{(x,y)}R_{x,y}$
"$a \le c$ or $b \le d$",
${}_{(3,6)}R_{(7,4)}$&&${}_{(7,4)}R_{(2,5)}$ but not ${}_{(3,6)}R_{(2,5)}$, .Hence R is reflexive but not transitive.

75. Ans:b
U/S: Represent number of elements in U which are not in S. S=9, U/S=14
Lets assume each number as the node of a graph G. Degree of each vertices in S is known to be 3 but degree of vertices in U/S is unknown.
By definition of handshaking theorem : "There are even number of vertices of odd degree".
In the graph G , there are 9 vertices(part of S) of degree 3 and degree of 14 vertices(part of U/S) is not known. Hence there must be at-least 1 vertex in U/S which have odd degree to satisfy the handshaking theorem.Also all the vertices in U/S can not have odd degree because in that case there will be 23 vertices of odd degree , which is not valid as per handshaking theorem.Hence we can conclude that in U/S minimum 1 vertex should have odd degree and maximum 13 vertices should have odd degree . Option b is correct.

76. Ans:271
$N(3\cup5\cup7) = N(3)+N(5)+N(7)-N(3\cap5)-N(5\cap 7) - N(3\cap7) + N(3\cap5\cap7)$
$N(3) = \lfloor 500/3 \rfloor = 166$
$N(5) = \lfloor 500/5 \rfloor = 100$
$N(7) = \lfloor 500/7 \rfloor = 71$
$N(3\cap5) = \lfloor 500/15 \rfloor = 33$
$N(7\cap5) = \lfloor 500/35 \rfloor = 14$
$N(3\cap7) = \lfloor 500/21 \rfloor = 23$
$N(3\cap5\cap7) = \lfloor 500/105 \rfloor = 4$
$N(3\cup5\cup7) = 166+100+71-33-14-23+4 = 271$

77. Ans:d
Set of rational numbers are countable infinite .
Set of natural number are countable so N^2(Number of functions from {0,1} to N) is also countable but powerset(N) is not countable .All the subsets of natural numbers are uncountable but finite subset of natural numbers are countable.
Number of function from N to {0,1} are 2^N, which are uncountable . Hence option d is correct.