

GATEFREAKS

GATE/NET/PSU

COMPUTER SCIENCE

**Discrete Mathematics and Theory of
Computation**

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June 13, 2018

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Gatefreaks

Gatefreaks

1

Regular languages and Finite Automaton

Gatefreaks

1. Regular expression a^+b denotes the set:
 (A) $\{a\}$
 (B) $\{\epsilon, a, b\}$
 (C) $\{a, b\}$
 (D) None of these

UGCNET2005-II(june)

2. Which of the following strings is in the language defined by grammar $S \rightarrow 0A, A \rightarrow 1A/0A/1$
 (A) 01100
 (B) 00101
 (C) 10011
 (D) 11111

UGCNET2006-II(june)

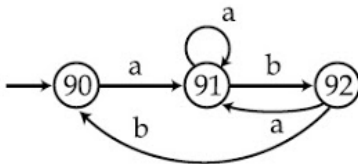
3. The logic of pumping lemma is a good example of:
 (A) pigeon hole principle
 (B) recursion
 (C) divide and conquer technique
 (D) iteration

UGCNET2006-II(june)

4. Which of the regular expressions corresponds to this grammar ?
 $S \rightarrow AB / AS, A \rightarrow a / aA, B \rightarrow b$
 (A) aa^*b^+
 (B) aa^*b
 (C) $(ab)^*$
 (D) $a(ab)^*$

UGCNET2006-II(dec.)

5. The following deterministic finite automata recognizes:



nizes:

- (A) Set of all strings containing 'ab'
 (B) Set of all strings containing 'aab'
 (C) Set of all strings ending in 'abab'
 (D) None of the above

UGCNET2007-II(jun.)

6. The regular expression given below describes:
 $r = (1 + 01)^*(0 + \lambda)$
 (A) Set of all string not containing '11'
 (B) Set of all string not containing '00'
 (C) Set of all string containing '01'
 (D) Set of all string ending in '0'

UGCNET2007-II(jun.)

7. Which of the following language is regular?
 (A) $L = \{a^n b^n | n \geq 1\}$
 (B) $L = \{a^n b^m c^n d^m | n, m \geq 1\}$
 (C) $L = \{a^n b^m | n, m \geq 1\}$
 (D) $L = \{a^n b^m c^n | n, m \geq 1\}$

UGCNET2007-II(jun.)

8. My Lafter Machin (MLM) recognizes the following strings :

- (i) a
 (ii) aba
 (iii) abaabaaba
 (iv) abaabaabaabaabaabaabaabaaba

Using this as an information, how would you compare the following regular expressions ?

- (i) $(aba)^{3^x}$
 (ii) $a.(baa)^{3^x-1}.ba$
 (iii) $ab.(aab)^{3^x-1}.a$
 (A) (ii) and (iii) are same, (i) is different.
 (B) (ii) and (iii) are not same.
 (C) (i), (ii) and (iii) are different.
 (D) (i), (ii) and (iii) are same.

UGCNET2010-II(Jun)

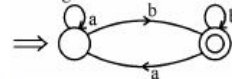
9. Which of the following is the most general phase structured grammar ?
 (A) Regular
 (B) Context-sensitive
 (C) Context free
 (D) None of the above

UGCNET2010-II(Jun)

10. Consider the regular expression $(a + b)(a + b) \dots (a + b)$ (n-times). The minimum number of states in finite automaton that recognizes the language represented by this regular expression contains
 (A) n states
 (B) $n + 1$ states
 (C) $n + 2$ states
 (D) 2^n states

UGCNET2012-III(JUN)

11. The regular expression for the following DFA is



- (A) $ab^*(b + aa^*b)^*$
 (B) $a^*b(b + aa^*b)^*$
 (C) $a^*b(b^* + aa^*b)$
 (D) $a^*b(b^* + aa^*b)^*$

UGCNET2012-III(JUN)

12. The number of bit strings of length eight that will either start with a 1 bit or end with two bits 00 shall be
 (A) 32
 (B) 64
 (C) 128
 (D) 160

UGCNET2012-II(Dec)

13. Let L be set accepted by a non-deterministic finite automaton. The number of states in non-deterministic finite automaton is $|Q|$. The maximum number of states in equivalent finite automaton that accepts L

- is
 (A) $|Q|$
 (A) $2|Q|$
 (A) $2^{|Q|} - 1$
 (A) $2^{|Q|}$

UGCNET2012-II(Dec)

14. Which is not the correct statement ?
 (A) The class of regular sets is closed under homomorphisms.
 (B) The class of regular sets is not closed under inverse homomorphisms.
 (C) The class of regular sets is closed under quotient.
 (D) The class of regular sets is closed under substitution.

UGCNET2012-III(Dec)

15. Which of the following regular expression identities are true ?
 (A) $(r + s)^* = r^* s^*$
 (B) $(r + s)^* = r^* + s^*$
 (C) $(r + s)^* = (r^* s^*)^*$
 (D) $r^* s^* = r^* + s^*$

UGCNET2012-III(Dec)

16. The minimum number of states of the non-deterministic finite automaton which accepts the language $\{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$ is
 (A) 3
 (B) 4
 (C) 5
 (D) 6

UGCNET2012-III(Dec)

17. Given $L_1=L(a^*baa^*)$ and $L_2=L(ab^*)$. The regular expression corresponding to language $L_3 = L_1/L_2$ (right quotient) is given by
 (A) a^*b
 (B) a^*baa^*
 (C) a^*ba^*
 (D) None of the above

UGCNET2013-II(JUN)

18. Given a Non-deterministic Finite Automation (NFA) with states p and r as initial and final states respectively and transition table as given below :

	a	b
p	-	q
q	r	s
r	r	s
s	r	s

The minimum number of states required in Deterministic Finite Automation (DFA) equivalent to NFA is
 (A) 5

- (B) 4
 (C) 3
 (D) 2

UGCNET2013-II(JUN)

19. The regular grammar for the language $L= \{w \mid n_a(w) \text{ and } n_b(w) \text{ are both even, } w \in \{a,b\}^*\}$ is given by : (Assume, p, q, r and s are states)
 (A) $p \rightarrow aq \mid br \mid \lambda, q \rightarrow bs \mid apr \rightarrow as \mid bp, s \rightarrow ar \mid bq, p$ and s are initial and final states.
 (B) $p \rightarrow aq \mid br, q \rightarrow bs \mid apr \rightarrow as \mid bp, s \rightarrow ar \mid bq, p$ and s are initial and final states.
 (C) $p \rightarrow aq \mid br \mid \lambda, q \rightarrow bs \mid apr \rightarrow as \mid bp, s \rightarrow ar \mid bq$ p is both initial and final states.
 (D) $p \rightarrow aq \mid br, q \rightarrow bs \mid apr \rightarrow as \mid bp, s \rightarrow ar \mid bq$ p is both initial and final states

UGCNET2014-II(Jun)

20. Let L be any language. Define even (W) as the strings obtained by extracting from W the letters in the even-numbered positions and $\text{even}(L) = \{\text{even}(W) \mid W \in L\}$. We define another language Chop (L) by removing the two leftmost symbols of every string in L given by $\text{Chop}(L) = \{W \mid \exists W \in L, \text{ with } |W| \geq 2\}$. If L is regular language then
 (A) $\text{even}(L)$ is regular and $\text{Chop}(L)$ is not regular
 (B) Both $\text{even}(L)$ and $\text{Chop}(L)$ are regular
 (C) $\text{Even}(L)$ is not regular and $\text{Chop}(L)$ is regular
 (D) Both $\text{even}(L)$ and $\text{Chop}(L)$ are not regular

UGCNET2014-III(Jun)

21. Given two languages :
 $L_1 = \{(ab)^n a^k \mid n > k, k \geq 0\}$
 $L_2 = \{a^n b^m \mid n \neq m\}$
 Using pumping lemma for regular language, it can be shown that
 (A) L_1 is regular and L_2 is not regular.
 (B) L_1 is not regular and L_2 is regular.
 (C) L_1 is regular and L_2 is regular.
 (D) L_1 is not regular and L_2 is not regular.

UGCNET2014-III(Dec)

22. Regular expression for the complement of language $L = \{a^n b^m \mid n \geq 4, m \leq 3\}$ is
 (A) $(a + b)^* ba(a + b)^*$
 (B) $a^* bbb^* b^*$
 (C) $(\lambda + a + aa + aaa)b^* + (a + b)^* ba(a + b)^*$
 (D) None of the above

UGCNET2014-III(Dec)

23. Minimal deterministic finite automaton for the language $L = \{0^n \mid n \geq 0, n \neq 4\}$ will have:
 (A) 1 final state among 5 states
 (B) 4 final states among 5 states
 (C) 1 final state among 6 states

(D)5 final state among 6 states
UGCNET2015-III(jun)

UGCNET2016-III(aug.)

24. The regular expression corresponding to the language L where $L = \{x \in \{0,1\}^* \mid x \text{ ends with } 1 \text{ and does not contain substring } 00\}$ is
 (A) $(1+01)^* (10+01)$
 (B) $(1+01)^* 01$
 (C) $(1+01)^* (1+01)$
 (D) $(10+01)^* 01$

UGCNET2015-III(jun)

25. The transition function for the language $L = \{w \mid n_a(w) \text{ and } n_b(w) \text{ are both odd}\}$ is given by:
 $\delta(q_0, a) = q_1 ; \delta(q_0, b) = q_2$
 $\delta(q_1, a) = q_0 ; \delta(q_1, b) = q_3$
 $\delta(q_2, a) = q_3 ; \delta(q_2, b) = q_0$
 $\delta(q_3, a) = q_2 ; \delta(q_3, b) = q_1$
 The initial and final states of the automata are
 (A) q_0 and q_0 respectively
 (B) q_0 and q_1 respectively
 (C) q_0 and q_2 respectively
 (D) q_0 and q_3 respectively

UGCNET2015-III(jun)

26. There are exactly..... different finite automata with three states x, y and z over the alphabet a, b where x is always the start state.
 (A) 64 (B) 256 (C) 1024 (D) 5832

UGCNET2015-III(dec.)

27. The number of strings of length 4 that are generated by the regular expression $(0^+1^+|2^+3^+)^*$, where | is an alternation character and +,* are quantification characters, is :
 (A) 08
 (B) 09
 (C) 10
 (D) 12

UGCNET2016-II(aug.)

28. The regular grammar for the language $L = \{a^n b^m \mid n + m \text{ is even}\}$ is given by
 (A) $S \rightarrow S_1 | S_2$
 $S_1 \rightarrow aS_1 | A_1$
 $A_1 \rightarrow bA_1 | \lambda$
 $S_2 \rightarrow aaS_2 | A_2$
 $A_2 \rightarrow bA_2 | \lambda$
 (B) $S \rightarrow S_1 | S_2$
 $S_1 \rightarrow aS_1 | aA_1$
 $S_2 \rightarrow aaS_2 | A_2$
 $A_1 \rightarrow bA_1 | \lambda$
 $A_2 \rightarrow bA_2 | \lambda$
 (C) $S \rightarrow S_1 | S_2$
 $S_1 \rightarrow aaaS_1 | aA_1$
 $S_2 \rightarrow aaS_2 | A_2$
 $A_1 \rightarrow bA_1 | \lambda$
 $A_2 \rightarrow bA_2 | \lambda$
 (D) $S \rightarrow S_1 | S_2$
 $S_1 \rightarrow aaS_1 | A_1$
 $S_2 \rightarrow aaS_2 | aA_2$
 $A_1 \rightarrow bbA_1 | \lambda$
 $A_2 \rightarrow bbA_2 | b$

29. Let $\Sigma = a, b$ and language $L = \{aa, bb\}$. Then, the complement of L is
 (A) $\{\lambda, a, b, ab, ba\} \cup \{w \in \{a, b\}^* \mid |w| > 3\}$
 (B) $\{a, b, ab, ba\} \cup \{w \in \{a, b\}^* \mid |w| \geq 3\}$
 (C) $\{w \in \{a, b\}^* \mid |w| > 3\} \cup \{a, b, ab, ba\}$
 (D) $\{\lambda, a, b, ab, ba\} \cup \{w \in \{a, b\}^* \mid |w| \geq 3\}$

UGCNET2016-III(aug.)

30. Consider the following identities for regular expressions :
 (a) $(r + s)^* = (s + r)^*$
 (b) $(r^*)^* = r^*$
 (c) $(r^* s^*)^* = (r + s)^*$
 Which of the above identities are true ?
 (A) (a) and (b) only
 (B) (b) and (c) only
 (C) (c) and (a) only
 (D) (a), (b) and (c)

UGCNET2016-III(aug.)

31. Given the following two languages :
 $L_1 = \{uww^R v \mid u, v, w \in (a, b)^+\}$
 $L_2 = \{uww^R v \mid u, v, w \in (a, b)^+, |u| \geq |v|\}$
 Which of the following is correct ?

- (A) L_1 is regular language and L_2 is not regular language.
 (B) L_1 is not regular language and L_2 is regular language.
 (C) Both L_1 and L_2 are regular languages.
 (D) Both L_1 and L_2 are not regular languages.

UGCNET2016-III(aug.)

32. The number of strings of length 4 that are generated by the regular expression $(0| \epsilon)1 + 2 * (3| \epsilon)$, where | is an alternation character, +, * are quantification characters, and ϵ is the null string, is:
 (A) 08
 (B) 10
 (C) 11
 (D) 12

UGCNET2016-II(jun.)

33. Which of the following strings would match the regular expression : $p + [3 - 5]^* [xyz]^*$?
 I. p443y
 II. p6y
 III. 3xyz
 IV. p35z
 V. p353535x
 VI. ppp5
 (A) I, III and VI only
 (B) IV, V and VI only
 (C) II, IV and V only
 (D) I, IV and V only

UGCNET2016-II(dec.)

34. Consider the languages $L_1 = \phi$ and $L_2 = \{1\}$. Which one of the following represents $L_1^* \cup L_2^* L_1^*$?
- (A) $\{\in\}$
 - (B) $\{\in, 1\}$
 - (C) ϕ
 - (D) 1^*

UGCNET2016-III(dec.)

35. Which of the following regular expressions, each describing a language of binary numbers (MSB to LSB) that represents non-negative decimal values, does not include even values ? (A) $0^*1^+0^*1^*$
- (B) $0^*1^*0^+1^*$
 - (C) $0^*1^*0^*1^+$
 - (D) $0^+1^*0^*1^*$
- Where $\{+, *\}$ are quantification characters.

UGCNET2017-II(Nov.)

36. The logic of pumping lemma is an example of
- (A) iteration
 - (B) recursion
 - (C) the divide and conquer principle
 - (D) the pigeon - hole principle

UGCNET2017-III(Nov.)

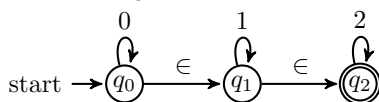
37. Pumping lemma for regular language is generally used for proving :
- (A) whether two given regular expressions are equivalent
 - (B) a given grammar is ambiguous
 - (C) a given grammar is regular
 - (D) a given grammar is not regular

UGCNET2017-III(Nov.)

38. Finite state machine can recognize language generated by
- (A) Only context free grammar
 - (B) Only context sensitive grammar
 - (C) Only regular grammar
 - (D) any unambiguous grammar

UGCNET2017-III(Nov.)

39. What are the final states of the DFA generated from the following NFA?



- (A) q_0, q_1, q_2
- (B) $[q_0, q_1], [q_0, q_2], []$
- (C) $q_0, [q_1, q_2]$
- (D) $[q_0, q_1], q_2$

ISRO 2013

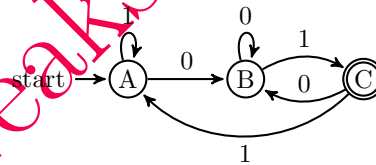
40. The number of states required by a Finite State Machine, to simulate the behavior of a computer with a memory capable of storing 'm' words, each of length 'n' bits is?
- (A) $m \times 2^n$
 - (B) 2^{m+n}
 - (C) 2^{mn}
 - (D) $m + n$

ISRO 2014

41. How many states are there in a minimum state deterministic finite automaton accepting the language $L = \{w \mid w \in \{0, 1\}^*, \text{ number of 0's is divisible by 2 and number of 1's is divisible by 5, respectively }\}$?
- (A) 7
 - (B) 9
 - (C) 10
 - (D) 11

ISRO 2014

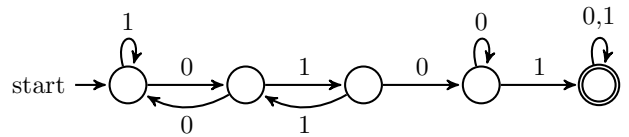
42. The following Finite Automaton recognizes which of the given languages?



- (A) $\{1, 0\}^* \{01\}$
- (B) $\{1, 0\}^* \{1\}$
- (C) $\{1\} \{1, 0\}^* \{1\}$
- (D) $1^* 0^* \{0, 1\}$

ISRO 2014

43. Consider the following Deterministic Finite Automaton M.



Let S denote the set of eight bit strings whose second, third, sixth and seventh bits are 1. The number of strings in S that are accepted by M is

- (A) 0
- (B) 1
- (C) 2
- (D) 3

ISRO 2014

44. Let R_1 and R_2 be regular sets defined over the alphabet, then
- (A) $R_1 \cap R_2$ is not regular
 - (B) $R_1 \cup R_2$ is not regular
 - (C) $\Sigma^* - R_1$ is regular
 - (D) R_1^* is not regular

ISRO 2015

45. Let $L = \{w \in (0 + 1)^* \mid w \text{ has even number of 1's}\}$, i.e. L is the set of all bit strings with even number of 1's. Which one of the regular expression below represents L ?
- (A) $(0^*10^*1)^*$
 - (B) $0^*(10^*10^*)^*$
 - (C) $0^*(10^*1^*)^*0^*$
 - (D) $0^*1(10^*1)^*10^*$

ISRO 2016

46. Context-free languages and regular languages are both closed under the operation (s) of :
- (A) Union
 - (B) Intersection
 - (C) Concatenation
 - (D) Complementation

GATE1989,1 MARK

47. Choose the correct alternatives (More than one may be correct).
Let R_1 and R_2 be regular sets defined over the alphabet Σ Then:
- (A) $R_1 \cap R_2$ is not regular.
 - (B) $R_1 \cup R_2$ is regular.
 - (C) $\Sigma^* - R_1$ is regular.
 - (D) R_1^* is not regular.

GATE1990,2 MARKS

48. Choose the correct alternatives (more than one may be correct) and write the corresponding letters only:
Let $r = 1(1 + 0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true?
- (A) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$
 - (B) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$
 - (C) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$
 - (D) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$
 - (E) None of the above

GATE1991,2 MARKS

49. Which of the following regular expression identities is/are TRUE?
- (A) $r(*) = r^*$
 - (B) $(r^*s^*)^* = (r + s)^*$
 - (C) $(r + s)^* = r^* + s^*$
 - (D) $r^*s^* = r^* + s^*$

GATE 1992,2 MARKS

50. In some programming language, an identifier is permitted to be a letter followed by any number of letters or digits. If L and D denote the sets of letters and digits respectively, which of the following expressions defines an identifier?

- (A) $(L + D)^+$
- (B) $(L.D)^*$
- (C) $L(L + D)^*$
- (D) $L(L.D)^*$

GATE 1995,1 MARK

51. A finite state machine with the following state table has a single input x and a single out z .

present state	next state, z	
	x=1	x=0
A	D, 0	B, 0
B	B, 1	C, 1
C	B, 0	D, 1
D	B, 1	C, 0

If the initial state is unknown, then the shortest input sequence to reach the final state C is:

- (A) 01
- (B) 10
- (C) 101
- (D) 110

GATE 1995,2 MARKS

52. Which two of the following four regular expressions are equivalent? (ϵ is the empty string).

- i: $(00)^*(\epsilon + 0)$
- ii: $(00)^*$
- iii: 0^*
- iv: $0(00)^*$

- (A) (i) and (ii)
- (B) (ii) and (iii)
- (C) (i) and (iii)
- (D) (iii) and (iv)

GATE 1996,2 MARKS

53. Let $L \subseteq \Sigma^*$ where $\Sigma = \{a, b\}$. Which of the following is true?

- (A) $L = \{x \mid x \text{ has an equal number of a's and b's}\}$ is regular
- (B) $L = \{a^n b^n \mid n \geq 1\}$ is regular
- (C) $L = \{x \mid x \text{ has more number of a's than b's}\}$ is regular
- (D) $L = \{a^m b^n \mid m \geq 1, n \geq 1\}$ is regular

GATE 1996,2 MARKS

54. Consider the following state table for a sequential machine. The number of states in the minimized machine will be

present state	Input	
	1	0
A	D, 0	B, 1
B	A, 0	C, 1
C	A, 0	B, 1
D	A, 1	C, 1
	Next state	output

- (A) 4
(B) 3
(C) 2
(D) 1

GATE 1996,2 MARKS

55. Which one of the following regular expressions over $\{0,1\}$ denotes the set of all strings not containing 100 as substring?
(A) $0^*(1+0)^*$
(B) 0^*1010^*
(C) $0^*1^*01^*$
(D) $0^*(10+1)^*$

GATE 1997,2 MARKS

56. If the regular set A is represented by $A = (01+1)^*$ and the regular set B is represented by $B = ((01)^*1^*)^*$, which of the following is true?
(A) $A \subset B$
(B) $B \subset A$
(C) A and B are incomparable
(D) $A = B$

GATE 1998,1 MARK

57. The string 1101 does not belong to the set represented by
(A) $110^*(0+1)$
(B) $1(0+1)^*101$
(C) $(10)^*(01)^*(00+11)^*$
(D) $(00+(11)^*0)^*$

GATE 1998,1 MARK

58. Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite automaton accepting L is
(A) 2
(B) 5
(C) 8
(D) 3

GATE 1998,2 MARKS

59. Which of the following statements is false?
(A) Every finite subset of a non-regular set is regular
(B) Every subset of a regular set is regular
(C) Every finite subset of a regular set is regular
(D) The intersection of two regular sets is regular

GATE 1998,2 MARKS

60. Consider the regular expression $(0+1)(0+1)\dots n$ times. The minimum state finite automaton that recognizes the language represented by this regular expression contains
(A) n states
(B) $n+1$ states

- (C) $n+2$ states
(D) None of the above

GATE 1999,1 MARK

61. Let S and T be languages over $\Sigma = \{a,b\}$ represented by the regular expressions $(a+b^*)^*$ and $(a+b)^*$, respectively. Which of the following is true?
(A) $S \subset T$
(B) $T \subset S$
(C) $S = T$
(D) $S \cap T = \phi$

GATE 2000,1 MARK

62. What can be said about a regular language L over $\Sigma = \{a\}$ whose minimal finite state automaton has two states?
(A) L must be $\{a^n | n \text{ is odd}\}$
(B) L must be $\{a^n | n \text{ is even}\}$
(C) L must be $\{a^n | n \geq 0\}$
(D) Either L must be $\{a^n | n \text{ is odd}\}$, or L must be $\{a^n | n \text{ is even}\}$

GATE 2000,2 MARKS

63. Consider the following two statements:
 $S1 : \{0^{2n} | n \geq 1\}$ is a regular language
 $S2 : \{0^m 1^n 0^{m+n} | m \geq 1 \text{ and } n \geq 1\}$ is a regular language
Which of the following statement is correct?
(A) Only $S1$ is correct
(B) Only $S2$ is correct
(C) Both $S1$ and $S2$ are correct
(D) None of $S1$ and $S2$ is correct

GATE 2001,1 MARK

64. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA at least
(A) N^2
(B) 2^N
(C) $2N$
(D) $N!$

GATE 2001,1 MARK

65. Consider a DFA over $\Sigma = \{a,b\}$ accepting all strings which have number of a 's divisible by 6 and number of b 's divisible by 8. What is the minimum number of states that the DFA will have?
(A) 8
(B) 14
(C) 15
(D) 48

GATE 2001,2 MARKS

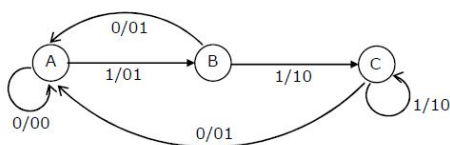
66. Consider the following languages:
 $L1 = \{ww \mid w \in \{a, b\}^*\}$
 $L2 = \{ww^R \mid w \in \{a, b\}^*, w^R \text{ is the reverse of } w\}$
 $L3 = \{0^{2^i} \mid i \text{ is an integer}\}$
 $L4 = \{0^{i^2} \mid i \text{ is an integer}\}$

Which of the languages are regular?

- (A) Only L1 and L2
- (B) Only L2, L3 and L4
- (C) Only L3 and L4
- (D) Only L3

GATE 2001,2 MARKS

67. The finite state machine described by the following state diagram with A as starting state, where an arc label is $\frac{x}{y}$ and x stands for 1-bit input and y stands for 2-bit output



- (A) Outputs the sum of the present and the previous bits of the input.
- (B) Outputs 01 whenever the input sequence contains 11
- (C) Outputs 00 whenever the input sequence contains 10
- (D) None of the above

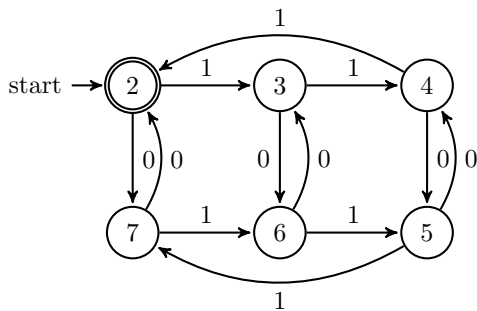
GATE 2002,2 MARKS

68. The smallest finite automaton which accepts the language $\{x \mid \text{length of } x \text{ is divisible by } 3\}$ has

- (A) 2 states
- (B) 3 states
- (C) 4 states
- (D) 5 states

GATE 2002,2 MARKS

69. The following finite state machine accepts all those binary strings in which the number of 1s and 0s are respectively



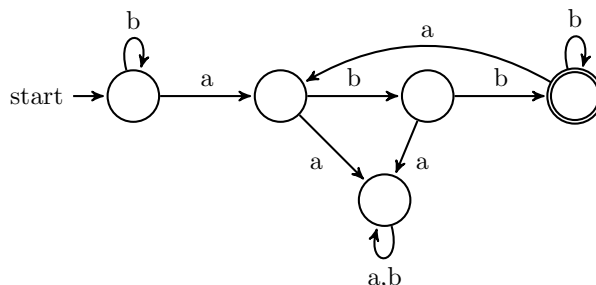
divisible by 3 and 2

- (B) odd and even
- (C) even and odd
- (D) divisible by 2 and 3

(A)

GATE 2004,2 MARKS

70. Consider the machine M:

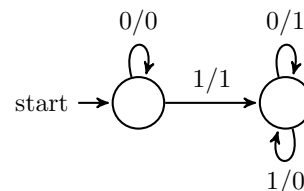


The language recognized by M is:

- (A) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by exactly two } b\text{'s}\}$
- (B) $\{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is followed by at least two } b\text{'s}\}$
- (C) $\{w \in \{a, b\}^* \mid w \text{ contains the substring 'abb'}\}$
- (D) $\{w \in \{a, b\}^* \mid w \text{ does not contain 'aa' as a substring}\}$

GATE 2005,2 MARK

71. The following diagram represents a finite state machine which takes as input a binary number from the least significant bit.



Which one of the following is TRUE?

- (A) It computes 1's complement of the input number
- (B) It computes 2's complement of the input number
- (C) It increments the input number
- (D) It decrements the input number

GATE 2005,2 MARK

72. Which of the following is TRUE?

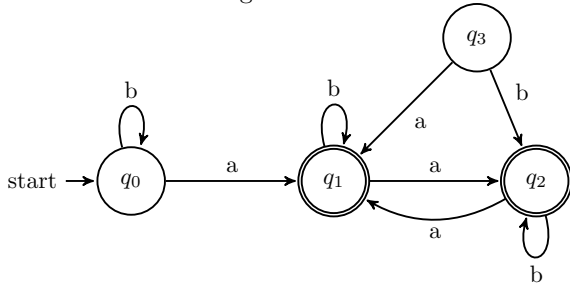
- (A) Every subset of a regular set is regular.
- (B) Every finite subset of a non-regular set is regular.
- (C) The union of two non-regular sets is not regular.
- (D) Infinite union of finite sets is regular

GATE 2007,1 MARK

73. A minimum state deterministic finite automaton accepting the language $L = \{w | w \in \{0, 1\}^*, \text{ number of 0s and 1s in } w \text{ are divisible by 3 and 5, respectively}\}$ has
- (A) 15 states
 - (B) 11 states
 - (C) 10 states
 - (D) 9 states

GATE 2007,2 MARKS

74. Consider the following Finite State Automaton:



A. The language accepted by this automaton is given by the regular expression

- (A) $b^*ab^*ab^*ab^*$
- (B) $(a+b)^*$
- (C) $b^*a(a+b)^*$
- (D) $b^*ab^*ab^*$

GATE 2007,2 MARKS

B. The minimum state automaton equivalent to the above FSA has the following number of states

- (A) 1
- (B) 2
- (C) 3
- (D) 4

GATE 2007,2 MARKS

75. Given below are two finite state automata (\rightarrow indicates the start state and F indicates a final state)

Y:

	a	b
\rightarrow 1	1	2
2(F)	2	1

Z:

	a	b
\rightarrow 1	2	2
2(F)	1	1

Which of the following represents the product automaton $Z \times Y$?

(A)

	a	b
\rightarrow P	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(B)

	a	b
\rightarrow P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(C)

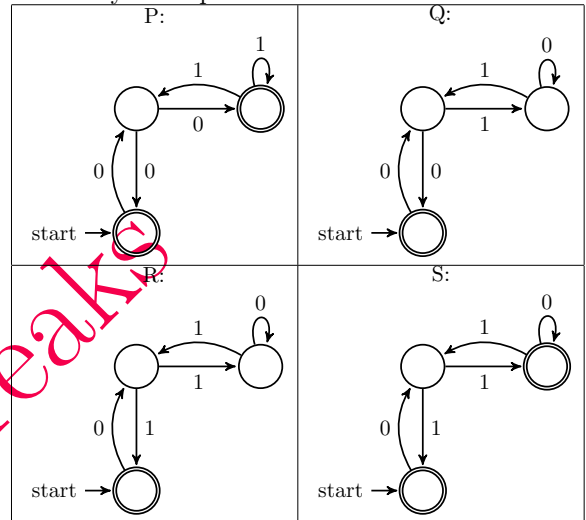
	a	b
\rightarrow P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(D)

	a	b
\rightarrow P	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

GATE 2008,2 MARKS

76. Match the following NFAs with the regular expressions they correspond to



- 1. $\epsilon + 0(01^*1 + 00)^*01^*$
- 2. $\epsilon + 0(10^*1 + 00)^*0$
- 3. $\epsilon + 0(01^*1 + 10)^*1$
- 4. $\epsilon + 0(01^*1 + 00)^*10^*$

- (A) P-2, Q-1, R-3, S-4
- (B) P-1, Q-3, R-2, S-4
- (C) P-1, Q-2, R-3, S-4
- (D) P-3, Q-2, R-1, S-4

GATE 2008,2 MARKS

77. Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression: $(0+1)^*0(0+1)^*0(0+1)^*?$

- (A) The set of all strings containing the substring 00.
- (B) The set of all strings containing at most two 0's.
- (C) The set of all strings containing at least two 0's.
- (D) The set of all strings that begin and end with either 0 or 1.

GATE 2009,1 MARK

78. Given the following state table of an FSM with two states A and B, one input and one output:

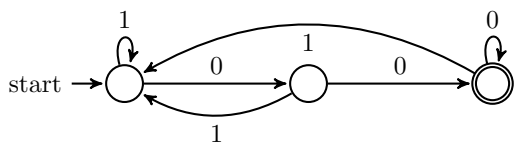
Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If

the initial state is A = 0, B=0, what is the minimum length of an input string which will take the machine to the state A=0, B=1 with Output=1?

- (A) 3
- (B) 4
- (C) 5
- (D) 6

GATE 2009,2 MARKS



79.

The above DFA accepts the set of all strings over {0, 1} that

- (A) begin either with 0 or 1
- (B) end with 0
- (C) end with 00
- (D) contain the substring 00.

GATE 2009,2 MARKS

80. Which one of the following is FALSE?

- (A) There is unique minimal DFA for every regular language
- (B) Every NFA can be converted to an equivalent PDA.
- (C) Complement of every context-free language is recursive.
- (D) Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

GATE 2009,1 MARK

81. Let $L = w \epsilon(0 + 1)^* | w$ has even number of 1s, i. e. L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L?

- (A) $(0^*10^*1)^*$
- (B) $0^*(10^*10^*)^*$
- (C) $0^*(10^*1^*)^*0^*$
- (D) $0^*1(10^*1)^*10^*$

GATE 2010,2 MARKS

82. Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

- (A) n - 1

- (B) n
- (C) n + 1
- (D) 2^{n-1}

GATE 2010,2 MARKS

83. Definition of a language L with alphabet $\{a\}$ is given as following.

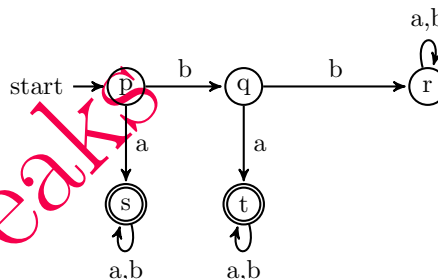
$$L = \{a^{nk} | k > 0, \text{ and } n \text{ is a positive integer constant.}\}$$

What is the minimum number of states needed in a DFA to recognize L?

- (A) k+1
- (B) n+1
- (C) 2^{n+1}
- (D) 2^{k+1}

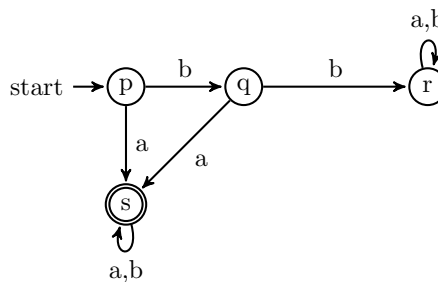
GATE 2011,2 MARKS

84. A deterministic finite automation(DFA) D with alphabet $\Sigma = \{a,b\}$ is given below

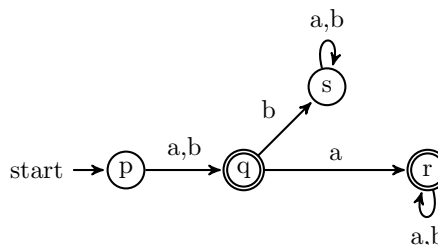


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

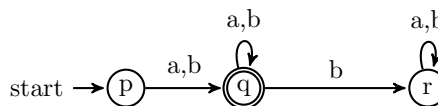
(A)



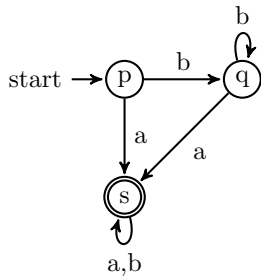
(B)



(C)



(D)

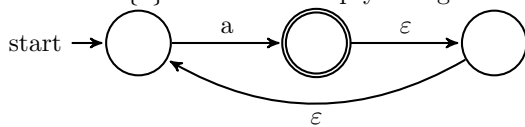


GATE 2011,2 MARKS

85. Given the language $L = \{ ab, aa, baa \}$, which of the following strings are in L^* ?
- 1) abaabaaabaa
 - 2) aaaabaaaa
 - 3) baaaaabaaaab
 - 4) baaaaabaa
- (A) 1,2 and 3
 (B) 2,3 and 4
 (C) 1,2 and 4
 (D) 1,3 and 4

GATE 2012,1 MARK

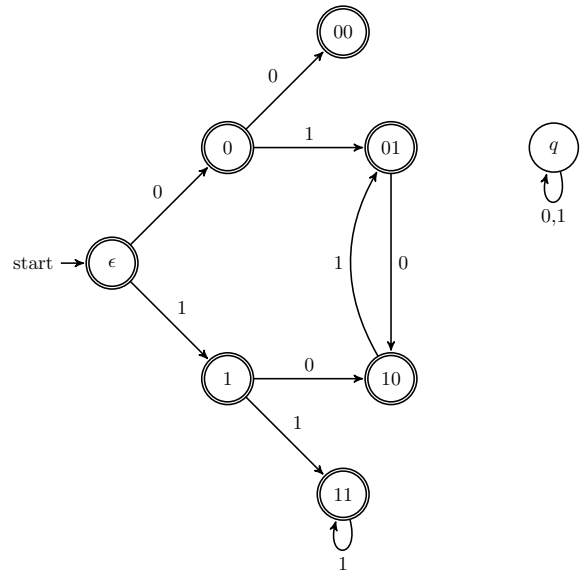
86. What is the complement of the language accepted by the NFA show below?
 Assume $\Sigma = \{a\}$ and ϵ is the empty string.



- (A) \emptyset
 (B) $\{ \epsilon \}$
 (C) a^*
 (d) $\{a, \epsilon\}$

GATE 2012,1 MARK

87. Consider the set of strings on $\{0,1\}$ in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

(A)

	00	01	10	11	q
00	1	0			
01				1	
10					
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

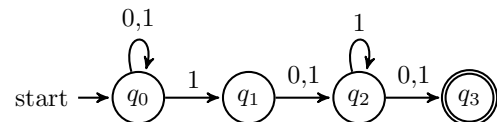
	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(D)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

GATE 2012,2 MARKS

88. Consider the finite automaton in the following figure.

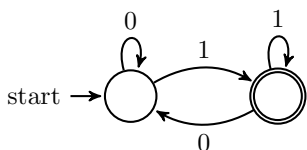


What is the set of reachable states for the input string 0011?

- (A) $\{q_0, q_1, q_2\}$
 (B) $\{q_0, q_1\}$
 (C) $\{q_0, q_1, q_2, q_3\}$
 (D) $\{q_3\}$

GATE 2014-I,1 MARK

89. Which of the regular expressions given below represent the following DFA ?



- I) $0^* 1 (1 + 00^* 1)^*$
- II) $0^* 1^* 1 + 11^* 0^* 1$
- III) $(0 + 1)^* 1$
- (A) I and II only
- (C) II and III only
- (B) I and III only
- (D) I, II, and III

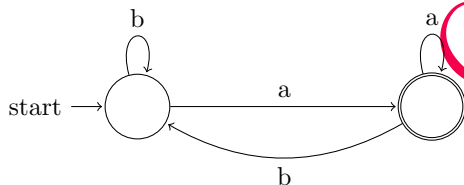
GATE 2014-I,2 MARKS

90. Let $L_1 = \{w \in \{0, 1\}^* | w \text{ has at least as many occurrences of } (110)\text{'s as } (011)\text{'s}\}$. Let $L_2 = \{w \in \{0, 1\}^* | w \text{ has at least as many occurrences of } (000)\text{'s as } (111)\text{'s}\}$. Which one of the following is TRUE?

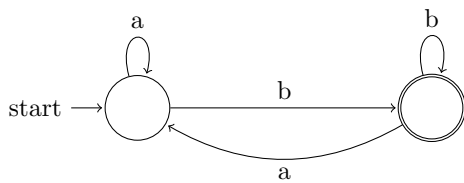
- (A) L_1 is regular but not L_2
- (B) L_2 is regular but not L_1
- (C) Both L_1 and L_2 are regular
- (D) Neither L_1 nor L_2 are regular

GATE 2014-II,2 MARKS

91. M:



N:



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is_____.

GATE 2015-I,2 MARKS

92. Consider the alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the set of strings X_0, X_1 , and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 , and X_2 are related as follows.

$$X_0 = 1X_1$$

$$X_1 = 0X_1 + 1X_2$$

$$X_2 = 0X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0 ?

- (A) $10(0^* + (10)^*)1$
- (B) $10(0^* + (10)^*)^*1$
- (C) $1(0+10)^*1$
- (D) $10(0+10)^*1 + 110(0+10)^*1$

GATE 2015-II,2 MARKS

93. The number of states in the minimal deterministic finite automaton corresponding to the regular expression $(0 + 1)^*(10)$ is

GATE 2015-II,2 MARKS

94. Let L be the language represented by the regular expression $\sum^* 0011 \sum^* = \{0, 1\}$.

What is the minimum number of states in a DFA that recognizes \bar{L} (complement of L)?

- (A) 4
- (B) 5
- (C) 6
- (D) 8

GATE 2015-III,1 MARK

95. Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

- (A) $(0 + 1)^* 0011(0 + 1)^* + (0 + 1)^* 1100(0 + 1)^*$
- (B) $(0 + 1)^* (00(0 + 1)^* 11 + 11(0 + 1)^* 00)(0 + 1)^*$
- (C) $(0 + 1)^* 00(0 + 1)^* + (0 + 1)^* 11(0 + 1)^*$
- (D) $00(0 + 1)^* 11 + 11(0 + 1)^* 00$

GATE 2016-I,1 MARKS

96. The number of states in the minimum sized DFA that accepts the language defined by the regular expression $(0 + 1)^*(0 + 1)(0 + 1)^*$ is_____.

GATE 2016-II,1 MARKS

97. Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1 | \epsilon$

Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2 | \epsilon$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which one of the following is TRUE?

- (A) Both P and Q are true
- (B) P is true and Q is false
- (C) P is false and Q is true
- (D) Both P and Q are false

GATE 2016-II,1 MARKS

98. Consider the following two statements:

I. If all states of an NFA are accepting states then the

language accepted by the NFA is Σ^* .

II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Which one of the following is CORRECT?

- (A) Only I is true
- (B) Only II is true
- (C) Both I and II are true
- (D) Both I and II are false

GATE 2016-II,2 MARKS

99. Consider the language L given by the regular expression $(a + b)^*b(a + b)$ over the alphabet a,b. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is.....

GATE 2017-I,1 MARK

100. Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ϵ -NFA whose transition table is given below:

δ	ϵ	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
$\{q_2\}$	$\{q_0\}$	\emptyset	\emptyset
q_3	\emptyset	\emptyset	$\{q_2\}$

Then $\hat{\delta}(q_2, aba)$ is

- (A) \emptyset
- (B) $\{q_0, q_1, q_3\}$
- (C) $\{q_0, q_1, q_2\}$
- (D) $\{q_0, q_2, q_3\}$

GATE 2017-II,2 MARKS

101. Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N . Which one of the following is necessarily true?

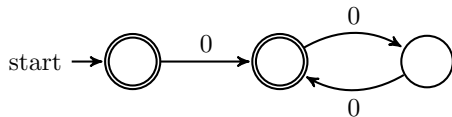
- (A) $k \geq 2^n$
- (B) $k \geq n$
- (C) $k \leq n^2$
- (D) $k \leq 2^n$

GATE 2018,1 MARK

102. Given a language L , define L^i as follows:

$$L^0 = \{\epsilon\}$$

$$L^i = L^{i-1} \bullet L \text{ for all } i > 0$$



The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet O) accepted by the following automaton. The order of L_1 is..... GATE 2018,2 MARKS

Solutions

1. Ans:c

Finite automata corresponding to given regular expression accepts a or b but not empty string .Hence option c is correct.

2. Ans:b

S is the start symbol , $S \rightarrow 0A$ specifies that every string should start with 0, hence option c and d can be ruled out.

A can generate $(0 + 1)^*1$, hence every string should end with 1, hence option a can be omitted. Correct answer is option b as:

$S \rightarrow 0A \rightarrow 00A \rightarrow 001A \rightarrow 0010A \rightarrow 00101$.

3. Ans:a

The proofs of pumping lemma typically require counting arguments such as the pigeonhole principle. Hence, the logic of pumping lemma is a good example of the pigeonhole principle.

4. Ans:b

$S \rightarrow AB|AS$

$A \rightarrow a|aA$

Language corresponding to A is aa^*

Language corresponding to S is $aa^*b|(aa^*)^*b$

Hence option b is correct. Moreover, Given grammar will generate strings having exactly one occurrence of b (ending with b). Hence option a , c, d are not valid .

5. Ans:d

Language accepted by an automaton depends on the accepting state . Given automaton doesn't have any accepting state, So we can't say anything about the language accepted by given machine . Hence option d is correct .

6. Ans:b

Given regular expression corresponds to the language having set of all string not having consecutive 0's, Hence option b is correct. Lets check other options :

A:False, because 111 is accepted by given regular expression.

C:False, because 0001 is not accepted by given regular expression.

D:False, because 1 is accepted by given regular expression .

7. Ans:c

A:DCFL, as there is comparison between n and m, hence one stack space is required.

B:CSL, As 2 stacks are required.

C:Regular, as there is no comparison between m and n , Regular expression will be a^+b^+

D:DCFL, as there is comparison between n and m, hence one stack space is required.

8. Ans:d

i) $(aba)^3$ For $x = 0$, it generates (aba)

For $x = 1$, it generates $(abaabaaba)$

For $x = 2$, it generates $(abaabaabaabaabaabaabaabaaba)$

ii) $a.(baa)^{3^x-1}.ba$

For $x = 0$, $a.(baa)^{3^0-1}.ba$ generates (aba)

For $x = 1$, $a.(baa)^{3^1-1}.ba$ generates $(abaabaaba)$

For $x = 2$, $a.(baa)^{3^2-1}.ba$ generates $(abaabaabaabaabaabaabaabaaba)$

iii) $ab.(aab)^{3^x-1}.a$

For $x = 0$, $ab.(aab)^{3^0-1}.a$ generates (aba)

For $x = 1$, $ab.(aab)^{3^1-1}.a$ generates $(abaabaaba)$

For $x = 2$, $ab.(aab)^{3^2-1}.a$ generates $(abaabaabaabaabaabaabaabaaba)$

Hence i, ii, and iii are equivalent.

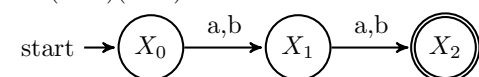
9. Ans:b

Context sensitive grammars are phase structured grammars.

10. Ans:b

Lets consider the smaller case with $n=2$

$L=(a+b)(a+b)$



Number of states=3,

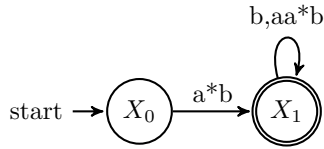
For NFA , number of states =n+1

for DFA, number of states =n+2 (we need to include dead-state also)

As minimum number of states are asked , so option b is correct.

11. Ans:b

Using state elimination method :



Option b is correct.

12. Ans:d

L: string of length 8 starting with 1 i.e $1 \times \times \times \times \times \times$, where \times can be 0/1.

$N(L)$: Number of strings satisfying L = $2^7 = 128$

K: string of length 8 ending with 00 i.e. $\times \times \times \times \times 00$, where \times can be 0/1.

$N(K)$ = Number of strings satisfying K = $2^6 = 64$

$L \cap K$: string of length 8 starting with 1 and end with 00 i.e $1 \times \times \times \times 00$.

$N(L \cap K) = 2^5 = 32$

$L \cup K$: string of length 8 either starting with 1 or ending with 00.

$N(L \cup K) = N(L) + N(K) - N(L \cap K)$
 $= 128 + 64 - 32 = 160$.

13. Ans:d

A NFA has $|Q|$ states. A state in equivalent DFA will be the subset of the set of states NFA. Hence maximum number of states will be $2^{|Q|}$.

14. Ans:b

15. Ans:c

$(r + s)^* = (r^* + s^*)^* = (r^*s^*)^* = (r + s^*)^* = (r^* + s)^*$. So option c is correct.

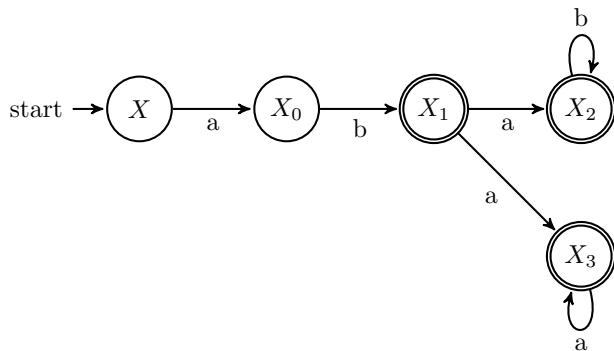
16. Ans:c

$L1 = \{abab^n | n \geq 0\} = \{aba, abab, ababb, \dots\}$

$L2 = \{aba^n | n \geq 0\} = \{ab, aba, abaa, \dots\}$

$L1 \cup L2 = \{ab, aba, abaa, abab, ababb, \dots\}$

Finite automata :



17. Ans:c

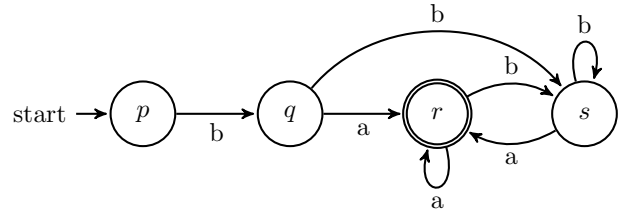
$L1 = a^*baa^* = \{ba, aba, abaa, abaaa^*, aa^*baaa^* \dots\}$

$L2 = \{ab^*\} = \{a, ab, abb^* \dots\}$

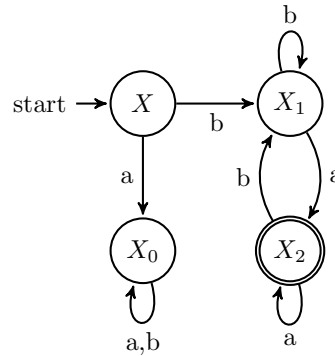
$L1/L2 = a^*ba^* = \{b, ab, aba, abaa, a^*baa^* \dots\}$

18. Ans:b

NFA corresponding to given table.

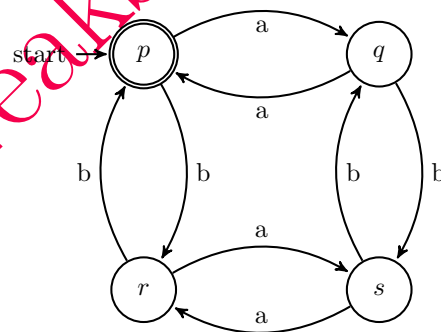


Minimized DFA :



19. Ans:c

Automata corresponding to the given grammar is given as :



20. Ans:b

$W = A_1A_2A_3A_4 \dots$ So, $even(W) = A_2A_4 \dots$. We can design the automata for this, so it is regular.

1. First design the automata M for W, with q_0 as initial state.

2. Make the initial state q_0 a non-initial state and introduce new initial state q_0' .

3. Make ϵ -transitions from q_0' to all states of M.

4. For each input we move two states in the same way we can design the NFA for chop(L). So both are regular.

21. Ans:d

As there is comparisons, so stack (PDA) is required. Both languages are not regular.

22. Ans:d

$\overline{L_1} = \{a^n b^m | n \geq 4, m \leq 3\}$

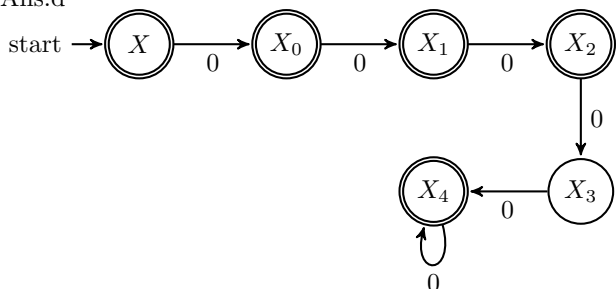
$= \{a^n b^m | n < 4\} \cup \{a^n b^m | m > 3\}$

$\{a^n b^m | n < 4\} = \{b^* + ab^* + aab^* + aaab^*\}$

$\{a^n b^m | m > 3\} = \{a^* bbb^*\}$

Hence regular expression for $\overline{L_1}$ is $(\epsilon + a + aa + aaa)b^* + a^* bbb^*$.

23. Ans:d

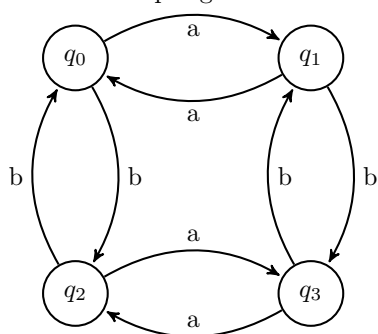


24. Ans:c

A:Incorrect, as 1001 is possible which shouldn't be.
 B:Incorrect, as 1 is not possible.
 C:Correct. D:Incorrect, as 1001 is possible which shouldn't be.

25. Ans:d

Automata as per given transitions is given as:



Given automaton should accept { ab, ba, aaab, aaba, abaa, bbab..... }
 So q₀ should be initial state and q₃ should be final state.

26. Ans:d

1. Total number of DFA possible with K states and R input symbols = $K * K^{(K*R)} * 2^K$ where, K*R defines total number of Transitions
 Number of transition functions between K states and (K*R) transitions = $K^{(K*R)}$
 2^K Defines possibilities for no. of final states .
 Here, initial state is given ,So
 $1 * 3^{(3 * 2)} * 2^3 = 5832$.

27. Ans:c

There are 10 strings of length 4 ,given as:
 0001, 0111, 0011, 0101, 0123, 2323, 2333, 2223, 2233, 2301.

28. Ans:d

A and C are not possible as string ab can be generated by given grammar.
 B is not possible , as string aabaab can be generated , which is not in form $a^n b^n$.

29. Ans:d

\bar{L} : Contains all the strings of length >2 , and all the string of length ≤ 2 except {aa,bb}
 Option A and C are not possible as strings of length

3 are not possible.

Option B is not possible as λ is not included.

30. Ans:d

.

31. Ans:a

L_1 is regular ,Regular expression for $L_1 = (a + b)^+(aa + bb)(a + b)^+$
 L_2 is not regular ,because of comparisons between u and v.

32. Ans:d

starting with 0 and ending with 3 = 0123,0113,
 starting with 0 = 0122,0112,0111
 ending with 3 = 1223,1123,1113
 Not containing 0 and 3 = 1222,1122,1112,1111 Total 12 strings are possible .

33. Ans:d

$p + [3-5]^*[xyz] = p + (3+4+5)*(x+y+z)$
 II. $p6y$:Not possible ,as 6 is not in regular expression.
 III. $3xyz$:Not possible ,as Strings must start with p.
 VI. $ppp5$:Not possible ,as Strings must end with x, or y, or z.

34. Ans:d

$L_1 \cup L_2 \cup L_3 \cup L_4$
 $= \phi^* \cup 1^* \phi^*$
 $= \in \cup 1^* \in \{ \text{because } \in = \phi^* = \in \}$
 $= \in \cup 1^* \{ \text{because } \in * A = A \}$
 $= 1^* \{ \text{Because } 1^* \text{ itself contains } \in \}$.

35. Ans:c

In binary representation , LSB bit 1 represents the odd number and 0 represent the even numbers. In option C , LSB will always be 1.

36. Ans:d

The proofs of pumping lemma typically require counting arguments such as the pigeonhole principle. Hence, the logic of pumping lemma is a good example of the pigeonhole principle.

37. Ans:d

.

38. Ans:c

.

39. Ans:a

.

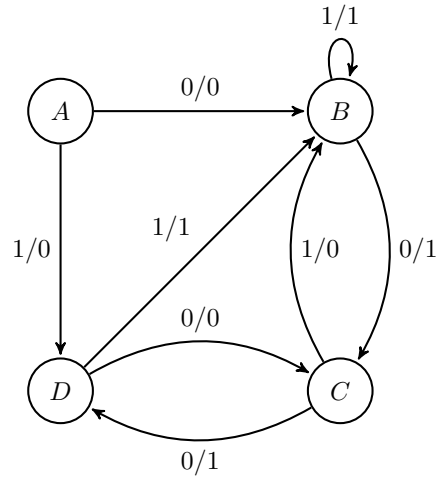
40. Ans:c

There are m words ,each of which are of n bits , So, total number of bits = $m * n$.
 We require separate state for each bit combination.
 So, no. of states = 2^{mn} .

41. Ans:c

Number of equivalence classes for (divisible by 2)#a=0,1 and (divisible by 5)#b=0,1,2,3,4
 Total number of states = $2 * 5 = 10$.

42. Ans:a
Given automaton accepts all the strings ending with 01. Hence option a is correct.
43. Ans:c
There are 2 strings accepted by M :
i)01110110
ii) 01110111.
44. Ans:c
By closure property, Regular languages are closed under Intersection , Union, Kleen closure(*) and Difference . So option c is correct.
45. Ans:b
A:Incorrect, Strings ending with 0 are not possible i.e 010010.
B:Correct.
C: Incorrect, 1* can generate any number of 1's.
D:Incorrect, strings with zero number of 1's can't be generated.
46. Ans:a,c
.
47. Ans:b,c
.
48. Ans:a,c
s : any numbers of 1's followed by one 0, for example 11110, 1111110....
r : 1 followed by universal set of 0 or 1, i.e., 1, 110, 101, 11010....
t : any numbers of 1 followed by only one 0, i.e., 0, 110, 11110, ..., zero number of 1's is also allowed.
So, All strings generated by s, can be generated by r $\Rightarrow L(s) \subseteq L(r)$ is true. All strings generated by s can be generated by t, $\Rightarrow L(s) \subseteq L(t)$.
49. Ans:b
A: Not correct, as $r^* = r \neq r^*$
B:Correct.
C:Not correct, As $(r + s)^*$ can generate all the string on r or s, but r^*s^* can't generate the combination of r and s , for example, string rsr can not be generated.
D:not correct, As LHS can generate rss but RHS can't.
.
50. Ans:c
L followed by L or D implies L,LL,LDL,LDD,LDLL.....
A:not correct , it specifies : "any combination of letters and digits (excluding NULL string)"
B, not correct, it implies : "each letter is followed by a digit(including NULL string)"
C, Correct.
D:Not correct, As LL combination is not possible.
51. Ans:b
Machine for given table :
Automata as per given transitions is given as:



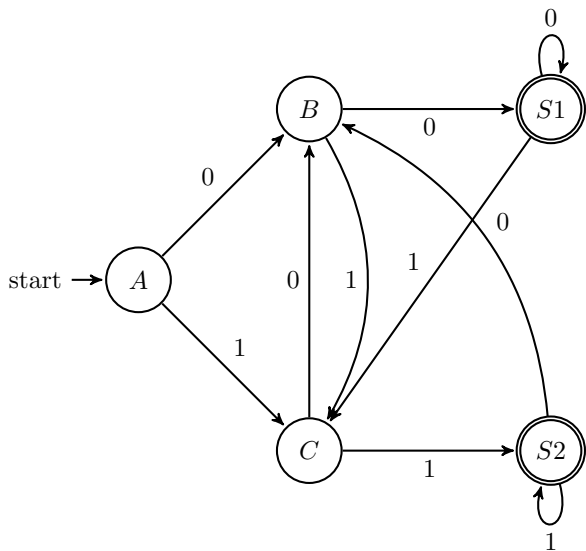
Here, shortest sequence will be 0 to reach C(if B is start state), but this is not given in the option . Next shortest string will be 10,(C is initial state) .

52. Ans:c
i: Generate any number of 0's(Including empty string).
ii: Generate even number of 0's (Including empty string)
iii: Generate any number of 0's (Including empty string)
iv: Generate odd number of 0's.
53. Ans:d
To make comparisons between number of a's and number of b's , we require stack(PDA), and fall under context free languages. Here in option a ,b and d ,we are comparing number of a's and b's so d is correct answer.
54. Ans:b
2 states M and N are equivalent if $\delta(M, a) = N$ and $\delta(N, a) = M$,
Also,
 $\{\delta(M, b) = \delta(N, b) | \forall b, b \neq a, b, a \in \Sigma^*\}$
Here B and C can be combined , So number of states =3.
55. Ans:d
Let,
 Σ^* : set of all strings on 0 or 1.
B: Set of all strings containing 100.
A:set of all strings not containing 100.
Hence, $A = \Sigma^* - B$
 $A = \{\epsilon, 1, 10, 11, 111, 101, \dots\}$
option a and b are not correct as 100 is the part of given regular expression.
option c is not correct because ϵ can not be generated by given regular expression.
Option d is correct.
56. Ans:d
 $(r+s)^* = (r^* + s^*)^* = (r^*s^*)^* = (r+s^*)^* = (r^* + s)^*$.

Here Both regular expressions generates all strings over $\{0,1\}$ where every 0 is followed by a 1.

57. Ans:c

58. Ans:b



59. Ans:b

Let $\Sigma = \{0, 1\}$

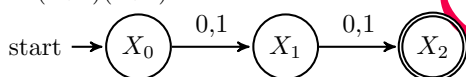
$0^n 1^n \subset \Sigma^*$

$0^n 1^n$ is not regular, so option b is correct.

60. Ans:b

Lets consider the smaller case with $N=2$

$L=(0+1)(0+1)$



Number of states=3,

For NFA, number of states =N+1

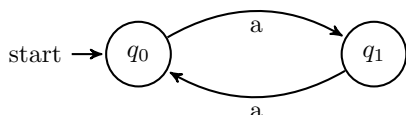
for DFA, number of states =N+2 (we need to include dead-state also)

As minimum number of states are asked, so option b is correct.

61. Ans:c

$(r+s)^* = (r^*+s^*)^* = (r^*s^*)^* = (r+s^*)^* = (r^*+s)^*$.

62. Ans:d



If q_1 is accepting state, then $\{a^n | n \text{ is odd}\}$,

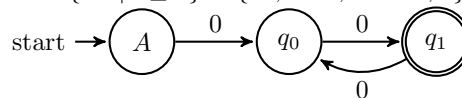
If q_0 is accepting state, then $\{a^n | n \text{ is even}\}$.

63. Ans:a

We can design DFA for S1 but not for S2. As there are comparisons in S2 so PDA is required.

DFA for S1 is given as:

$S1 = \{0^{2n} | n \geq 1\} = \{00, 0000, 000000, \dots\} = (aa)^+$



64. Ans:b

65. Ans:d

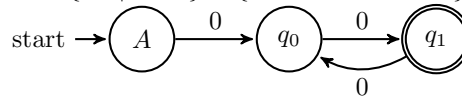
Number of equivalence classes for (divisible by 6) #a=0,1,2,3,4,5 and (divisible by 8) #b=0,1,2,3,4,5,6,7

Total number of states =6*8=48.

66. Ans:d

L1 and L4 are CSL, L2 is CFL, L3 is regular. DFA can be constructed for L3 as:

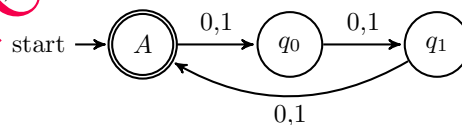
$S1 = \{0^{2n} | n \geq 1\} = \{00, 0000, 000000, \dots\} = (aa)^+$



67. Ans:c

68. Ans:b

Number of equivalence classes for (divisible by 3) #a=0,1,2. So 3 states are required.



69. Ans:a

A:Correct.

B:Incorrect, as string 111111 is also accepted by given DFA.

C:Incorrect, as string 111 is accepted by Given DFA.

D:Incorrect, Same as Option C.

70. Ans:b

A : Incorrect, As abbb is accepted.

C : Incorrect, since abba contains abb as substring, but is still not accepted.

D : Incorrect, since aba does not contain aa as substring, but is still not accepted.

71. Ans:b

In 2's complement algorithm, any number of 0's followed by a 1 will be same(starting from LSB) and then rest of bits are complemented. Given machine calculate 2's complement of given binary bits.

72. Ans:b

Let $\Sigma = \{0, 1\}$

A:Incorrect, as $\{0^n 1^n\} \subseteq \Sigma^*$

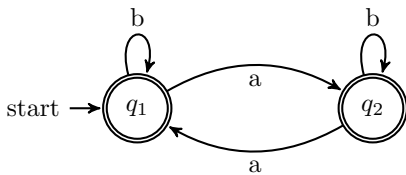
B:Correct, every finite set is regular.

C:Incorrect, As $\{0^m 1^n | m < n\} \cup \{0^m 1^n | m \geq n\} = 0^* 1^*$, which is regular.

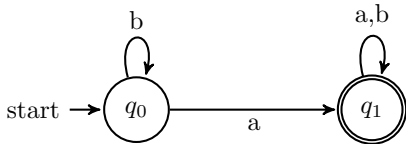
D:Incorrect, As regular languages are not closed under infinite union.

73. Ans:a
 Number of equivalence classes for (divisible by 3) #a=0,1,2 and (divisible by 5) #b=0,1,2,3,4
 Total number of states =3*5=15.

74. AnsA:c,AnsB:b
 Consider the following automata M:



all the strings over {0,1} are accepted by M, So regular expression for M will be (a+b)*.
 Given problem can be minimized as:



Regular expression :b*a(a+b)*.

75. Ans:n/a

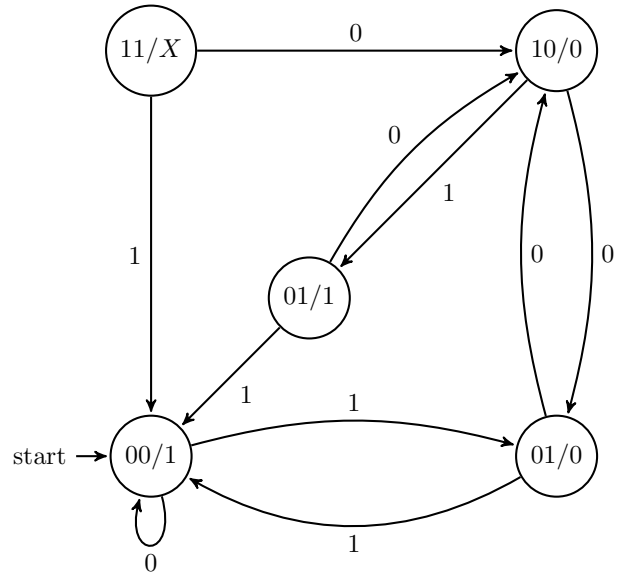
States	a	b
11	12	22
12	11	21
21	22	12
22 (F)	21	11

Let 11:P , 12:S, 21:Q, 22:R
 No option match.

76. Ans:c

77. Ans:c
 A:Incorrect, As 010 is also the part of given regular expression.
 B:Incorrect, As 000 is also the part of given regular expression.
 C:Correct.
 D:Incorrect, 1000 and 0001 is also the part of regular expression.

78. Ans:a
 Machine corresponding to the given table is :



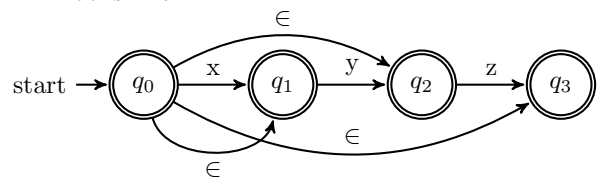
To reach 00 to state 01 with output 1 , 3 length input is required.

79. Ans:c
 A:False, as strings {101,0101} are not accepted by given DFA.
 B:False, as 10 is not accepted by given DFA.
 C:True.
 D:False, as 1001 is not accepted by given DFA.

80. Ans:d
 A:True, there is unique DFA for every regular language.
 B:True , Power of NFA and DFA is equal , so one can be converted to other.
 C:True , CFL is subset of recursive languages and recursive languages are closed under complement.
 D:False: Power of NPDA is more as compare to DPDA so conversion is not possible.

81. Ans:b
 A:Incorrect, Strings ending with 0 are not possible i.e 010010.
 B:Correct.
 C: Incorrect, 1* can generate any number of 1's.
 D:Incorrect, strings with zero number of 1's can't be generated.

82. Ans:c
 Lets consider a string(S) of size 3 ,xyz.
 Substrings of S are {ε,x,y,z,xy,yz,xyz}



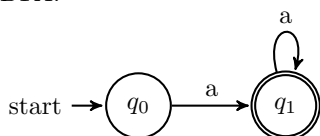
ε-NFA can be converted into the equivalent NFA with same number of states . Number of states for a string of size n will be n+1.

83. Ans:b

For $n = 1$

$$L = \{a^k | k > 0\} = \{a, aa, aaa, aaaa, \dots\} = a^+$$

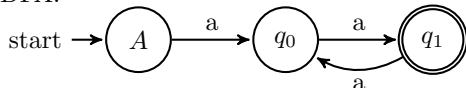
DFA:



for $n = 2$

$$L = \{a^{2k} | k > 0\} = \{aa, aaaa, aaaaaa, \dots\} = (aa)^+$$

DFA:



84. Ans:a

B:Incorrect, string b is accepted by given DFA but not by DFA D.

C:Incorrect, string b is accepted by given DFA but not by DFA D.

D: Incorrect,string bba is accepted by given DFA but not by DFA D.

85. Ans:c

Let $L = \{A, B, C\}$

$A = ab, B = aa, C = baa$

1) ABCAB

2) BBCB

3) Not possible

4) CBAB

86. Ans:b

Language(L) accepted by given automata is a^+

Complement of $L = \Sigma^* - a^+ = a^* - a^+ = \epsilon$.

87. Ans:d

Given automata should not accept 000. So, 00 when gets another 0 must go to a dead state, q.

88. Ans:a

$$1. \delta(q_0, 0011) \vdash \delta(q_0, 011) \vdash \delta(q_0, 11) \vdash \delta(q_0, 1) \vdash q_0$$

$$2. \delta(q_0, 0011) \vdash \delta(q_0, 011) \vdash \delta(q_0, 11) \vdash \delta(q_0, 1) \vdash q_1$$

$$3. \delta(q_0, 0011) \vdash \delta(q_0, 011) \vdash \delta(q_0, 11) \vdash \delta(q_1, 1) \vdash q_2.$$

89. Ans:b

Given automaton corresponds to set of all strings ending with 1.

I:Correct, As it accepts all the strings ending with 1.

II:Incorrect, As it doesn't accept 11011.

III:Correct, Same As I.

90. Ans:a

L1 is regular because we can't have two 011's in a string without a 110 or vice versa. For example String 11011011011011011 have five occurrence of 011 and 110. And we can have automata for this. So this is regular.

91. Ans:1

$L(M)$:Set of all strings ending with a

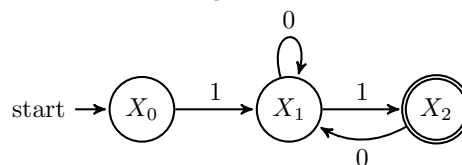
$L(N)$: Set of all strings ending with b.

$$L = L(M) \cap L(N) = (a+b)^*$$

DFA accepting $(a+b)^*$ require 1 state.

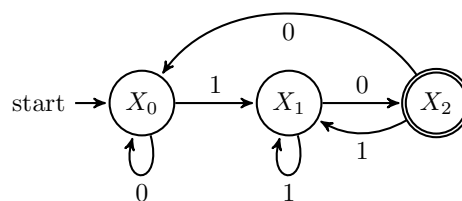
92. Ans:c

Automata for the given equations:



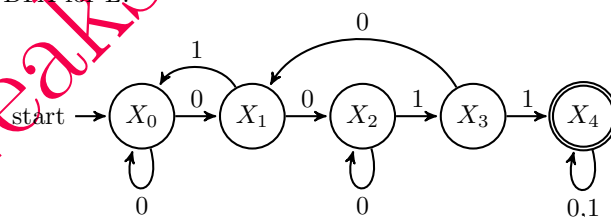
Regular expression : $1(0+10)^*1$.

93. Ans:3

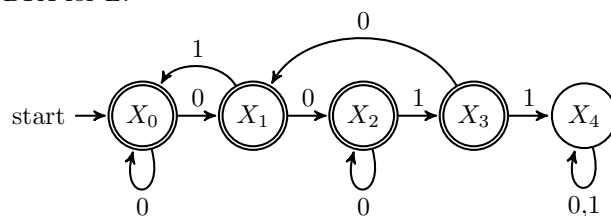


94. Ans:b

DEA for L :



DFA for \bar{L} :



95. Ans:b

Given language can be defined as:

$$\Sigma^* 00 \Sigma^* 11 \Sigma^* + \Sigma^* 11 \Sigma^* 00 \Sigma^*$$

$$(0+1)^* 00 (0+1)^* 11 (0+1)^* + (0+1)^* 11 (0+1)^* 00 (0+1)^*$$

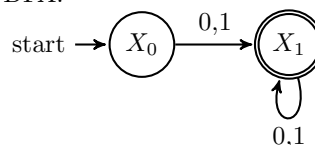
$$(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00) (0+1)^*$$

96. Ans:2

Given language accepts all the strings over $\{0,1\}$ of length ≥ 1

Regular expression can be written as $(0+1)^+$

DFA:



97. Ans:c

$$S_1 \rightarrow aS_1b | \epsilon$$

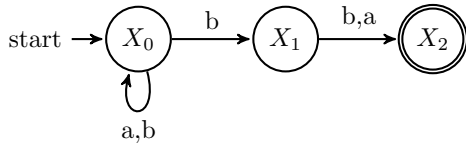
$L_1 = \{a^k b^k \mid k \geq 0\}$ is non-regular
 $S_2 \rightarrow abS_2 \mid \epsilon$
 $L_2 = \{(ab)^*\}$ is Regular .

98. Ans:b

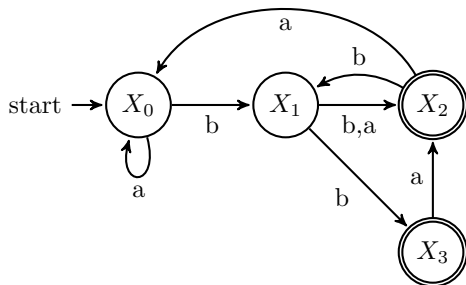
$S_1 \rightarrow aS_1 b \mid \epsilon$
 $L_1 = \{a^k b^k \mid k \geq 0\}$ is non-regular
 $S_2 \rightarrow abS_2 \mid \epsilon$
 $L_2 = \{(ab)^*\}$ is Regular .

99. Ans:4

NFA for the given regular expression:

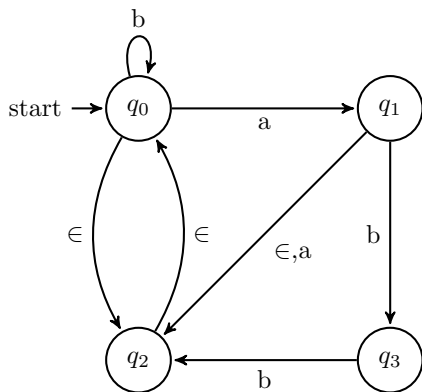


Above NFA can be converted into the equivalent DFA as:



100. Ans:c

ϵ -NFA for the given table :



$\hat{\delta}(q_2, aba) = \{q_0, q_1, q_2\}$.

101. Ans:d

.

102. Ans:2

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