GATEFREAKS

GATE/NET/PSU

COMPUTER SCIENCE

Design and Analysis of Algorithms

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Hashing

1. A hash table with ten buckets with one slot per bucket is shown in the following figure. The symbols S1 to S7 initially entered using a hashing function with linear probing.The maximum number of comparisons needed in searching an item that is not present is

0	S7
1	S1
2	
3	S4
4	S2
5	
6	S5
7	
8	S6
9	S3

(A)4
(B)5
(C)6
(D)3

GATE1989,1 MARK

2. Which of the following statements is true?
I:As the number of entries in a hash table increases, the number of collisions increases.
II:Recursive programs are efficient
III:The worst case complexity for Quicksort is $O(n^2)$
IV:Binary search using a linear linked list is efficient
(A)I and II
(B)II and III
(C)I and IV
(D)I and III

GATE 1995,2 MARKS

3. An advantage of chained hash table (external hashing) over the open addressing scheme is
(A)Worst case complexity of search operations is less
(B)Space used is less
(C)Deletion is easier
(D)None of the above

GATE 1996,2 MARKS

4. Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function x mod 10, which of the following statements are true?
i) 9679, 1989, 4199 hash to the same value
ii) 1471, 6171 has to the same value
iii) All elements hash to the same value
iv) Each element hashes to a different value
(A) i only
(B) ii only
(C) i and ii only
(D) iii or iv

GATE 2004,1 MARK

5. A hash table contains 10 buckets and uses linear probing to resolve collisions. The key values are integers and the hash function used is key % 10. If the values 43, 165, 62, 123, 142 are inserted in the table, in what location would the key value 142 be inserted?
(A) 2
(B) 3
(C) 4
(D) 6

GATE2005-IT,1 MARK

6. Consider a hash function that distributes keys uniformly. The hash table size is 20. After hashing of how many keys will the probability that any new key hashed collides with an existing one exceed 0.5.
(A) 5
(B) 6
(C) 7
(D) 10

GATE2007-IT,1 MARK

7. Consider a hash table of size 11 that uses open addressing with linear probing. Let h(k) = k mod 11 be the hash function used. A sequence of records with keys
43 36 92 87 11 4 71 13 14
is inserted into an initially empty hash table, the bins of which are indexed from zero to ten. What is the index of the bin into which the last record is inserted?
(A) 3
(B) 4
(C) 6
(D) 7

GATE2008-IT,2 MARKS

8. Consider a hash table of size seven, with starting index zero, and a hash function (3x+4)mod7.Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that - denotes an empty location in the table.
(A) 8, -, -, -, -, -, 10
(B) 1, 8, 10, -, -, -, 3
(C) 1, -, -, -, -, -, 3
(D) 1, 10, 8, -, -, -, 3

GATE 2007,2 MARKS

9. The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using

open addressing with hash function h(k) = k mod 10 and linear probing . What is the resultant hash table?

a)

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

b)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

c)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

d)

0	
1	
2	12,2
3	13,3,23
4	
5	5,15
6	
7	
8	18
9	

GATE 2009,2 MARKS

10. A hash table of length 10 uses open addressing with hash function h(k)=k mod10 , and linear probing. After inserting 6 values into an empty hash table,the table is as shown below

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

A. Which one of the Following choices gives a possible order in which the key values could have been inserted in the table?
(A) 46, 42, 34, 52, 23, 33
(B) 34, 42, 23, 52, 33, 46
(C) 46, 34, 42, 23, 52, 33
(D) 42, 46, 33, 23, 34, 52

GATE 2010,2 MARKS

B. How many different insertion sequences of the key values using the same hash Function and linear probing will result in the hash table shown above?
(A) 10
(B) 20
(C)
30
(D) 40

GATE 2010,2 MARKS

11. Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?
(A) $(97 \times 97 \times 97)/100^3$
(B) $(99 \times 98 \times 97)/100^3$ (C) $(97\times 96 \times 95)/100^3$

(D) $(97 \times 96 \times 95)/(3! \times 100^3)$

GATE 2014-III,2 MARKS

12. Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?
(A)$h(i) = i^2$mod 10
(B)$h(i) = i^3$mod 10
(C)$h(i) = (11 * i^2)$mod 10
(D)$h(i) = (12 * i^2)$mod 10

GATE 2015-II,2 MARKS

13. Given a has table T with 25 slots that stores 2000 elements,the load factor α for T is
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GATE 2015-III,1 MARK

14. A rule in a limited entry decision table is a
(A)row of the table consisting of condition entries
(B)row of the table consisting of action entries
(C)column of the table consisting of condition entries and corresponding action entries
(D)columns of the table consisting of conditions of the stub

ISRO 2007

15. A symbol table of length 152 is processing 25 entries at any instant. What is occupation density?
(A) 0.164
(B) 127
(C) 8.06
(D) 6.08

ISRO 2011

16. Consider a 13 element hash table for which f(key)=key mod 13 is used with integer keys. Assuming linear probing is used for collision resolution, at which location would the key 103 be inserted, if the keys 661, 182, 24 and 103 are inserted in that order?
(A) 0
(B) 1
(C) 11
(D) 12

ISRO 2014

17. A hash table with ten buckets with one slot per bucket is shown in the following figure. The symbols S1 to S7 initially entered using a hashing function with linear probing.The maximum number of comparisons needed in searching an item that is not present is

0	S7
1	S1
2	
3	S4
4	S2
5	
6	S5
7	
8	S6
9	S3

(A) 4
(B) 5
(C) 6
(D) 3

ISRO 2015

18. A Hash Function f defined as $f(key) = key$ mod 7. With linear probing while inserting the keys $37, 38, 72, 48, 98, 11, 56$ into a table indexed from 0, in which location key 11 will be stored (Count table index 0 as 0^{th} location)?
(A) 3
(B) 4
(C) 5
(D) 6

ISRO 2016

19. A hash function f defined as f (key) = key mod 13, with linear probing is used to insert keys 55, 58, 68, 91, 27,145. What will be the location of 79 ?
(A) 1
(B) 2
(C) 3
(D) 4

UGCNET2012-II(Dec)

20. Searching for an element in the hash table requires $O(1)$ time for the ....time, whereas for direct addressing it holds for the .... time.
(A) worst-case, average
(B) worst-case, worst-case
(C) average, worst-case
(D) best, average

UGCNET2014-II(Jun)

21. If h is chosen from a universal collection of hash functions and is used to hash n keys into a table of size m, where n ≤m, the expected number of collisions involving a particular key K is
(A) less than 1
(B) less than lg n
(C) greater than 1
(D) greater than lg n

UGCNET2014-III(Jun)

22. Consider a hash table of size m=100 and the hash function h(k)=floor (m(kA mod 1)) for $A = \frac{(\sqrt{5}-1)}{2} = 0.618033$. Compute the location to which the key k=123456 is placed in hash table
(A)77
(B)82
(C)88
(D)89

UGCNET2015-III(jun)

23. The hash function used in double hashing is of the form:
(A) $h(k, i) = (h_1(k) + h_2(k) + i) \, mod \, m$
(B) $h(k, i) = (h_1(k) + h_2(k) - i) \, mod \, m$
(C) $h(k, i) = (h_1(k) + i \, h_2(k)) \, mod \, m$
(D) $h(k, i) = (h_1(k) - i \, h_2(k)) \, mod \, m$

UGCNET2015-II(dec.)

24. Consider a hash table of size $m = 10000$, and the hash function $h(K) = floor(m(KA \bmod 1))$ for $A = (\sqrt{5} - 1)/2$. The key 123456 is mapped to location.....
(A) 46
(B) 41
(C) 43
(D) 48

UGCNET2016-III(aug.)

25. If h is chosen from a universal collection of hash functions and is used to hash n keys into a table of size m, where $n \leq m$, the expected number of collisions involving a particular key x is less than ....
(A) 1
(B) 1/n
(C) 1/m
(D) n/m

UGCNET2016-II(dec.)

Solutions

1. Ans:b
Lets assume no item has been deleted from the table so searching stops as soon as any empty slot appear. Number of comparison =size of cluster +1. Suppose we want to search for element S_8, which should be on the slot number 8. While searching S_8 cluster will be (S_6, S_3, S_7, S_1) . So number of comparisons will be 5.

2. Ans:d
I:True, Load factor $= \frac{\text{Number of hash table entries}}{\text{hash table size}}$,So load factor increases with number of entries in the hash table. Probability of collision increases as load factor increases .
II:False, Recursive programs are not efficient due to system and stack overhead.
III:True, In the worst case , time complexity of quick sort is $O(n^2)$
IV:False, Time complexity of searching in the linear linked list of size n ,using binary search is $O(n \log n)$.

3. Ans:c
I:False, because in worst case all elements could be mapped to a single slot so searching time will be O(n) in chaining .
II:False, because chaining require more space as compared to open addressing as some space is wasted in the form of links to next node.
III:True, because in chaining we can directly delete the node but in open addressing we need to insert marker element after deletion to keep track of deleted element.

4. Ans:c
I:True,because 9679,1989,4199 mapped to slot 9
II:True , because 1471 and 6171 mapped to slot 1
So option c is correct.

5. Ans:d
After insertion of all the keys , resultant hash table will looks like:

0	
1	
2	62
3	43
4	123
5	165
6	142
7	
8	
9	

6. Ans:b
Probability that there will be a collision after n hashes = 1−Probability that there was no collision in the first n hashes
$= 1 - 1.\frac{19}{20}.\frac{18}{20}\dots\frac{20-n+1}{20}.$
$0.5 < 1 - 1.\frac{19}{20}.\frac{18}{20}\dots\frac{20-n+1}{20}.$
$\Rightarrow \frac{19}{20}.\frac{18}{20}\dots\frac{20-n+1}{20} < 0.5.$
For $n = 6$, we get 0.43605. So, correct answer is b.

7. Ans:d

0	87
1	11
2	13
3	36
4	92
5	4
6	71
7	14
8	
9	
10	43

.

8. Ans:b
In closed hashing all the key values are stored in the table itself , In case of collision , rehashing technique is used . As in question rehashing technique is not mentioned so we can use simplest technique which is

linear probing(LP) .
1:$(3 \times 1 + 4)$mod7 =0
3:$(3 \times 3 + 4)$mod7 =6
8:$(3\times 8+4)$mod7 =0 , collision so using LP new hash value (0+1)mod7=1
10:$(3 \times 10 + 4)$mod7 =6 collision so using LP new hash value (6+1)mod7=0,⇒ (0+1)mod7 = 1, ⇒ (1+1)mod7=2

9. Ans:c
In the linear probing , if collision occur then key will be inserted on next empty bucket.
•12 mod 10=2
•18 mod 10=8
•13 mod 10=3
•2 mod 10=2 ,collision ⇒(2+1)mod 10=3, collision ⇒ (3+1)mod 10=4
•3 mod 10=3, collision ⇒(3+1)mod 10=4, collision⇒ (4+1) mod 10=5
•23 mod 10=3 ,collision ⇒(3+1)mod 10=4, collision⇒ (4+1) mod 10=5, collision⇒(5+1)mod 10=6
•5 mod 10=5, collision⇒ (5+1)mod 10=6, collision ⇒ (6+1)mod 10=7
•15 mod 10=5, collision⇒ (5+1)mod 10=6, collision ⇒(6+1)mod 10=7 collision⇒(7+1) mod 10=8,collision ⇒(8+1) mod 10=9

10. AnsA:c,AnsB:c
Hash table corresponding to given options can be represented as :

A)

0	
1	
2	42
3	52
4	34
5	23
6	46
7	33
8	
9	

C)

0	
1	
2	42
3	23
4	34
5	52
6	33
7	46
8	
9	

C)

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

D)

0	
1	
2	42
3	33
4	23
5	34
6	46
7	52
8	
9	

Clearly,option c is correct.

B:
33 occupy slot number 7 which means slot 3,4,5,6 have been filled before occurrence of 33. Also slot 2 should be filled before slot 6 . So 33 must be on the last position
Now among the remaining elements 52 must occur after {42,23,34}, So here we have 2 possibilities:
I:{42,23,34} ,52,46: i.e. 52 must follow {42,23,34},
II:{42,23,34,46},52: i.e. 52 must follow {42,23,34,46}
For the first case there are 3! permutations and for second case there are 4! permutations. So total number of sequences possible are 3!+4!=30

11. Ans:a
Here collision is resolved using chaining so for every key to be inserted ,there are 97 slots available.
Probability that key is inserted in any of 97 slots except first/second/third slots=$\frac{97}{100}$.
Probability for 3 insertions:$\frac{97}{100} \times \frac{97}{100} \times \frac{97}{100}$

12. Ans:b
A;False , because last digit in the squre of any number can not be 2,3,7,8 so these four slots will always be empty.
B:True , all the slots will be filled uniformly
C:False, same reason as of A.
D:False , as number is multiplied by 12(even number) so hash value will be a even number ,all odd numbered slots will be empty.

13. Ans:80
Load factor for a hash table=$\frac{\text{Table size}}{\text{Number of elements}}=\frac{2000}{25} = 80$

14. Ans:c
By the definition of decision table, Column of the table consisting of condition entries and corresponding action entries

15. Ans:a
Occupation density=(number of entries in the table)/ (Table size)=25/152=0.164

16. Ans:b

0	182
1	103
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	661
12	24

17. Ans:b
Lets assume no item has been deleted from the table so searching stops as soon as any empty slot appear. Number of comparison =size of cluster +1. Suppose we want to search for element S_8, which should be on the slot number 8. While searching S_8 cluster will be (S_6, S_3, S_7, S_1) . So number of comparisons will be 5.

18. Ans:c

Location	Key
0	98
1	56
2	37
3	38
4	72
5	11
6	48

19. Ans:n/a

0	91
1	27
2	145
3	55
4	68
5	79
6	58
7	
8	
9	
10	
11	
12	

20. Ans:c
.

21. Ans:a
onsider the definition of Universal hashing theorem: "If h is chosen from a universal collection of hash functions and is used to hash n keys into a table of size m, where n¡=m, the expected number of collisions involving a particular key x is less than 1"
Option a is correct.

22. Ans:c
h(k)=$\lfloor(100((123456 \times 0.618033) \text{ mod } 1))\rfloor$
=$\lfloor(100(76189.8820 \text{ mod } 1)\rfloor$
=$\lfloor(100(0.8820))\rfloor$
=88

23. Ans:c
By the definition of double hashing :
$h(i,k) = (h_1(k) + i(h_2(k))$ mod m
Option c is correct.

24. Ans:b
h(k)=$\lfloor(10000((123456 \times \frac{(\sqrt{5}-1)}{2}) \text{ mod } 1))\rfloor$
=$\lfloor(10000(76300.004115 \text{ mod } 1)\rfloor$
=$\lfloor(10000(0.004115))\rfloor$
=41

25. Ans:a
Consider the definition of Universal hashing theorem: "If h is chosen from a universal collection of hash functions and is used to hash n keys into a table of size m, where n¡=m, the expected number of collisions involving a particular key x is less than 1"
Option a is correct.